2017

(6th Semester)

PHYSICS

NINTH PAPER

(Mathematical Physics—II)

(Revised)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. *(a)* What are regular and irregular singularities of a differential equation? Explain them with examples. 1+1=2

(b) Solve the differential equation

$$2x^2 y'' - xy' + (1 - x^2)y = 0$$

by Frobenius method. 8

Or

Solve the wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

under the boundary conditions

$$y(0,\, t) = 0\,; \quad y(l,\, t) = 0$$

$$y(x,\, 0) = a \sin \frac{\pi x}{l}$$

$$\frac{\partial y}{\partial t}(x,\, 0) = 0$$

10

2. *(a)* Prove that $P_n(1) = 1$. 3

(b) Show that

$$\int_{-1}^{1} P_m(x)\, P_n(x)\, dx = \frac{2}{2n+1}\, \delta_{m,\, n}$$

7

Or

(a) For Hermite polynomials $H_n(x)$, show that

$$2x\, H_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$$

4

(b) Show that $J_n(x)$ is the coefficient of z^n in the expansion of $e^{x/2(z - \frac{1}{z})}$. 6

3. *(a)* Find the Fourier series representing

$$f(x) \quad x;\ 0 \quad x \quad 2$$ 5

(b) Find the finite Fourier sine and cosine transform of $\frac{{}^2u}{x^2}$, where u is a function of x and t for $0 \quad x \quad l,\ t \quad 0$. 2½×2=5

Or

(a) Define finite Fourier sine transform of a function. Hence find the Fourier sine transform of $F(x) \quad x$ such that $0 \quad x \quad 2$. 2+2=4

(b) Deduce the Fourier integral for a function. 6

4. *(a)* Define Laplace transform of a function. Find the Laplace transform of *(i)* e^{at} and *(ii)* $\sin at$. 2+2+2=6

(b) Using Laplace transform, solve the differential equation

$$ty\ (t) \quad y\ (t) \quad ty(t) \quad 0$$

under the condition that $y(0) \quad 1$ and $y(t)$ is bounded. 4

Or

(a) Using Laplace transform, evaluate

$$_0 t^2 e^{\ \ t} \sin t\, dt$$ 4

(b) Find the inverse Laplace transform of

$$\frac{1}{s^2 \quad 7s \quad 12}$$ 3

(c) Using inverse Laplace transform, show that

$$_0 \cos e^{\ \ x^2} dx \quad \frac{1}{2}\sqrt{\ }$$ 3

5. *(a)* What do you mean by a variable in FORTRAN? What are the different types of variables in FORTRAN? State the general rules for naming a variable in FORTRAN programming. 1+2+2=5

(b) Write a FORTRAN program to calculate the magnitude of

$$\vec{A} \quad A_1\hat{i} \quad A_2\hat{j} \quad A_3\hat{k}$$ 2

(c) Explain any three FORTRAN control statements with examples. 3

Or

(a) Write a FORTRAN program to evaluate a cosine series up to n terms. 4

(b) Write a FORTRAN program to find the slope and midpoint of a line. 4

(c) Find the value of K after the following program segment is executed : 2

```
K = 0
DO 10I = 5, 25, 3
   K = K + I
   IF (K.GT.12) GO TO 15
   10 CONTINUE
   15 K = 2 * K
```

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Subject Code : PHY/VI/09 (R)

Booklet No. A

Date Stamp ....

....

To be filled in by the Candidate

DEGREE 6th Semester
(Arts / Science / Commerce / ....) Exam., **2017**

Subject ....

Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

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DEGREE 6th Semester
(Arts / Science / Commerce / ....) Exam., **2017**

Roll No. ....

Regn. No. ....

Subject ....

Paper ....

Descriptive Type
Booklet No. B ....

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(6th Semester)

PHYSICS

NINTH PAPER

(Mathematical Physics—II)

(Revised)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. The differential equation of a circle having centre at the origin (0, 0) and radius r is

(a) $x^2 + y^2 = r^2$ ( )

(b) $x\,dx + y\,dy = 0$ ( )

(c) $y\,dx + x\,dy = 0$ ( )

(d) $x\,dx - y\,dy = 0$ ( )

2. Consider the differential equation $P_0 y'' + P_1 y' + P_2 y = 0$, where P_0, P_1 and P_2 are polynomials in x. The point $x = a$ is an ordinary point if

(a) P_0 does not vanish for $x = a$ ( )

(b) P_0 vanishes for $x = a$ ( )

(c) P_0, P_1 and P_2 vanish for $x = a$ ( )

(d) None of the above ( )

3. Legendre polynomial $P_0(x)$ is

(a) 1 ( )

(b) x ( )

(c) $3x^2 - 1$ ( )

(d) $1 - 2x$ ( )

4. For Hermite polynomials $H_n(x)$, $H_1(x)$ is given by

(a) 1 ( )

(b) $2x$ ( )

(c) 0 ( )

(d) x ( )

5. The Fourier series of an even function contains

(a) only the cosine terms ( )

(b) only the sine terms ( )

(c) both the sine and cosine terms ( )

(d) None of the above ( )

6. The function $f(x) = x^3$ in the range $-\pi < x < \pi$ is

(a) an odd function ( )

(b) an even function ( )

(c) a numeric function ( )

(d) a delta function ( )

7. The Laplace transform of $F(t) = 1$ is

(a) 1 ( )

(b) $\frac{1}{s}$, $s > 0$ ( )

(c) s, $s > 0$ ( )

(d) $\frac{1}{s^2}$, $s > 0$ ( )

8. The inverse Laplace transform of 1 is

(a) 1 ( )

(b) $\delta(t)$ ( )

(c) $\delta(t-1)$ ( )

(d) 0 ( )

9. Which of the following is a valid variable name in FORTRAN?

(a) $A*123$ ( )

(b) $123*A$ ( )

(c) $123\ A$ ( )

(d) $A123$ ( )

10. If $A = 3$, $B = 8$ and $C = 4$, then the value of D in the statement $D = 3*B / A*C - 4$ is

(a) 12 ( )

(b) 32 ( )

(c) 30 ( )

(d) 28 ( )

SECTION—II

(*Marks* : 15)

Answer the following questions : 3×5=15

1. Find the regular singular point of the differential equation

$$2x^2y'' + 3xy' + (x^2 - 4)y = 0$$

2. Show that $J_{-n}(x) = (-1)^n J_n(x)$, where n is a positive integer.

3. Find the inverse Fourier cosine transform of $e^{-\lambda n}$.

4. Find the Laplace transform of Bessel function $J_0(x)$.

5. Write a FORTRAN program that can be used to find the factorial of a positive integer.

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