2015

(6th Semester)

PHYSICS

NINTH PAPER

(Mathematical Physics—II)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. *(a)* Define gamma function [with argument n, denote it by $\Gamma(n)$]. Discuss the fundamental properties of gamma function. 1+5=6

(b) In connection with gamma function, prove the following : 2×2=4

(i) $\int_0^\infty e^{-\lambda x} x^{n-1} dx = \frac{\Gamma(n)}{\lambda^n}$ (λ, $n > 0$)

(ii) $\int_0^\infty e^{-x^2} x^{2n-1} dx = \frac{\Gamma(n)}{2}$ ($n > 0$)

(*Turn Over*)

Or

(a) From the definition of beta function [denoted by $\beta(m, n)$ for arguments m and n], show that

(i) $\beta(m, n) = \beta(n, m)$

(ii) $\int_0^{\frac{\pi}{2}} \cos^{2m-1}\theta \sin^{2n-1}\theta \, d\theta = \frac{\beta(m, n)}{2}$ 2+2=4

(b) Prove the following : 2×3=6

(i) $\int_0^{\frac{\pi}{2}} \sin^n \theta d\theta = \frac{\Gamma\left(\frac{n+1}{2}\right)\sqrt{\pi}}{2\Gamma\left(\frac{n}{2}+1\right)}$

(ii) $\beta(m+1, n) + \beta(m, n+1) = \beta(m, n)$

(iii) $\beta(m, n+1) = \frac{n}{m+n}\beta(m, n)$

2. *(a)* Explain Fourier series for a periodic function. State the conditions under which the Fourier series for a function converges to it. 2+2=4

(b) Find the Fourier series for the output of a half-wave rectifier. From the series, show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$$ 4+2=6

Or

(a) Deduce the Fourier integral for a function and express it in complex form. 4+2=6

(b) Define Fourier transform of a fucntion. Show that the Fourier transform of the function $f(t) = e^{-at^2}$ is $g(\omega) = \dfrac{e^{-\omega^2/4a}}{\sqrt{2a}}$. 2+2=4

3. *(a)* Define Laplace transform of a function. Find the Laplace transform of *(i)* x^n and *(ii)* $\cos ax$. 2+(2+2)=6

(b) Show that

$$L\{f^{(n)}(x)\} = s^n L\{f(x)\} - \sum_{r=0}^{n-1} s^{n-1-r} f^{(r)}(0)$$

where $L\{f(x)\}$ is the Laplace transform of $f(x)$ with respect to the kernel e^{-sx} $(s > 0)$ and $f^{(n)}(x) = \dfrac{d^n}{dx^n}(f(x))$. 4

Or

(a) Use Laplace transform to show that

$$\int_0^{\infty} \frac{\sin xt}{t}\, dt = \frac{\pi}{2}$$ 3

(b) Find the inverse Laplace transform of

$$F(s) = \frac{2s+1}{s^2 - 5s + 6}$$ 3

(c) Using Laplace transform, solve the differential equation $y'' + 2y' + y = e^{-x} \sin x$ with $y(0) = 0$, $y'(0) = 3$. Here $y' = \frac{dy}{dx}$ and $y'' = \frac{d^2 y}{dx^2}$. 4

4. (a) Define a cyclic group. Show that cyclic groups are Abelian. 1+2=3

(b) Show that the inverse of an element of a group is unique. 2

(c) Prove that the set of all unitary matrices of a given order forms a group under matrix multiplication. 3

(d) Define a subgroup of a group. What are the subgroups that a group possesses as trivial cases? 1+1=2

Or

Explain what you understand by symmetry operations and symmetry elements of a body. Discuss different types of symmetry operations and symmetry elements of a symmetric body. 2+2+6=10

5. (a) What do you mean by a variable in FORTRAN? What are different types of variables in FORTRAN? State the general rules for naming a variable in FORTRAN programing. 1+2+2=5

(b) Find the value of A after the following program segment in FORTRAN is executed : 2

```
      A = 0
      DO 10 I = 5, 25, 3
      A = A + I
      IF (A. GT. 15) GOTO 15
10    CONTINUE
15    A = A * A + 2 * A + 1
```

(c) Write a simple FORTRAN program to convert the temperature of a body in Celsius scale to that in Fahrenheit scale. 3

Or

(a) Explain DO-loops in FORTRAN. Write a FORTRAN program to evaluate the sum of first 100 natural numbers using a DO-loop. 3+3=6

(b) Consider the function

$$f(x) = \begin{cases} x^2 + \sin 2x & \text{if } x < 3\cdot 0 \\ 10\cdot 3 & \text{if } x = 3\cdot 0 \\ x^3 - \cos 3x & \text{if } x > 3\cdot 0 \end{cases}$$

Write a FORTRAN program to evaluate $f(x)$ for different values of x. What is the value of the function for $x = 1\cdot 5708$? 3+1=4

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2015

(6th Semester)

PHYSICS

NINTH PAPER

(Mathematical Physics—II)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. The value of $\Gamma(\frac{1}{2})\Gamma(-\frac{1}{2})$ is

(a) $2\sqrt{\pi}$ ( )

(b) $-2\sqrt{\pi}$ ( )

(c) $\frac{\sqrt{\pi}}{2}$ ( )

(d) $-\frac{\sqrt{\pi}}{2}$ ( )

2. The value of the integral $\int_0^\infty \frac{x^4 - x}{(1+x)^7} dx$ is

(a) 0 ( )

(b) 1 ( )

(c) 3 ( )

(d) ∞ ( )

3. The fourier series for an odd function contains

(a) only the cosine terms ( )

(b) only the sine terms ( )

(c) both the sine and cosine terms ( )

(d) only hyperbolic sine terms ( )

4. If $\delta(x)$ denotes the Dirac delta function, then

(a) $\int_{-\infty}^{\infty} \delta(x) f(x) dx = 0$ ( )

(b) $\int_{-\infty}^{\infty} \delta(x) f(x) dx = 1$ ( )

(c) $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$ ( )

(d) $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(x)$ ( )

5. If the kernel of the Laplace transform for a function of x is e^{-sx}, then the Laplace transform of 1 is

(a) 0 ( )

(b) 1 ( )

(c) s ( )

(d) $\frac{1}{s}$ ( )

6. If $F(s)$ is the Laplace transform of $f(x)$, then the inverse Laplace transform of $F\left(\frac{s}{a}\right)$ is

(a) $af(ax)$ ( )

(b) $\frac{1}{a}f(ax)$ ( )

(c) $af\left(\frac{x}{a}\right)$ ( )

(d) $\frac{1}{a}f\left(\frac{x}{a}\right)$ ( )

7. If G is a group of order g and H is its subgroup of order h, then which of the following is always true?

(a) g is an integral multiple of h ( )

(b) h is an integral multiple of g ( )

(c) g and h have no specific relation ( )

(d) g is an odd multiple of h ( )

8. The symmetry point group for ammonia (NH_3) molecule is

(a) C_3 ( )

(b) D_3 ( )

(c) $C_{3}v$ ( )

(d) $D_{3}h$ ( )

9. Which of the following is a valid variable name in FORTRAN?

(a) A * 123 ( )

(b) 123 * A ( )

(c) 123 A ( )

(d) A 123 ( )

10. In FORTRAN, the imaginary part of a complex number Z is obtained by

(a) IMAG (Z) ( )

(b) AIMAG (Z) ( )

(c) BIMAG (Z) ( )

(d) CIMAG (Z) ( )

SECTION—II

(*Marks* : 15)

Answer the following questions : 3×5=15

1. Prove that $\int_0^\infty \frac{x^{m-1}}{(ax+b)^{m+n}}\,dx = \frac{\Gamma(m)\Gamma(n)}{a^m b^n \Gamma(m+n)}$.

2. Express the Fourier series for a function in complex form

3. If $L\{f(x)\}$ is the Fourier transform of $f(x)$ with respect to the kernel e^{-sx}, then show that

$$L\left\{\frac{\cos ax - \cos bx}{b^2 - a^2}\right\} = \frac{s}{(s^2 + a^2)(s^2 + b^2)} \quad (a^2 \neq b^2)$$

4. Prove that no element can be common to any two distinct classes of a group.

8. Write a FORTRAN program that can be used to find the factorial of a positive integer.

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