2014

(6th Semester)

PHYSICS

NINTH PAPER

(Method of Mathematical Physics—II)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. *(a)* Using the definition of gamma function, find the value of $\Gamma(\frac{1}{2})$. Using this, obtain the values of $\Gamma(-\frac{1}{2})$ and $\Gamma(-\frac{3}{2})$. 4+2=6

   (b) Use the definition of Γ-function to evaluate the following integrals : 4

   (i) $\int_0^\infty e^{-ax} x^{m-1} \cos bx \, dx$

   (ii) $\int_0^\infty e^{-ax} x^{m-1} \sin bx \, dx$

Or

(a) Prove that

$$\Gamma(m)\,\Gamma\left(m+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2m-1}}\,\Gamma(2m)$$ 6

(b) Show that

$$\beta(m,\ n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$ 4

2. *(a)* Find the Fourier series expansion of the function defined as

$$f(x)=\begin{cases} x+\pi & \text{for } 0\le x\le\pi \\ -x-\pi & \text{for } -\pi\le x\le 0 \end{cases}$$

and $f(x+2\pi)=f(x)$. 6

(b) Find the finite cosine transforms of $f(x)$ in the interval $(0,\ \pi)$ if

(i) $f(x)=\dfrac{\pi}{3}-x+\dfrac{x^2}{2\pi}$

(ii) $f(x)=\sin ax$ 2+2=4

Or

(a) Find the Fourier cosine transform of $f(t)=e^{-pt}$, $p>0$. Hence evaluate

$$\int_0^{\infty}\frac{\cos\omega t}{p^2+\omega^2}\,d\omega$$ 3+2=5

(b) Prove that $\delta(ax) = \frac{1}{a}\delta(x);\ a > 0.$ 3

(c) Prove that $x\delta'(x) = -\delta(x)$. 2

3. *(a)* Find the Laplace transforms of *(i)* $\sin^2 t$ and *(ii)* $\cos^3 t$. 2+2=4

(b) Find the Laplace transform of sawtooth wave function

$$f(t) = \frac{at}{T} \text{ for } 0 < t < T \text{ and } f(t+T) = f(t)$$

3

(c) Find the inverse Laplace transforms of

$$\frac{1}{s^2 + a^2}$$

by residue method. 3

Or

(a) Use convolution theorem to find the functions whose Laplace transform is

$$\frac{s^2}{(s^2 + a^2)^2}$$

3

(b) Find the inverse Laplace transform of

$$f(s) = \frac{1}{s^2}\left(\frac{s-1}{s+1}\right)$$

4

(c) Using Laplace transform, evaluate the integral $\int_0^\infty t^2 e^{-t} \sin t\, dt$. 3

4. (a) Define a group and state the group axioms. 5

(b) Show that the three cube roots of unity form an Abelian group under multiplication. 3

(c) If the elements a, b and ab of a group G are each of order 2, then show that the group is Abelian. 2

Or

(a) What are conjugate elements? State and prove the properties of conjugate elements. 5

(b) If a and b be two elements of a group G and $ba = a^m b^n$ $\forall a, b \in G$, then show that the elements $a^m b^{n-2}$ and ab^{-1} have the same order. 3

(c) Show that the intersection of two subgroups of a group G is a subgroup of G. 2

5. (a) Suppose $P = 5$ and $Q = 8$. What will be the final value of P after executing the following program segment? 2

```
IF (3*P .LT. Q*2)
P=P+2
P=P+3
```

(b) Write a FORTRAN program to calculate the magnitude of $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$. 2

(c) What do you mean by FORTRAN control statements? Explain any three FORTRAN control statements with examples. 1+3=4

(d) Find the value of K after the following program segment is executed : 2

```
      K=0
      DO 10 I=5, 25, 3
      K=K+I
      IF (K .GT. 12) GO TO 15
10    CONTINUE
15    K=2*K
```

Or

(a) Write a DO loop to output—

(i) the odd integer between 1 and 49;

(ii) sum of the cubes of odd integers between 21 and 27. 2+2=4

(b) Write a FORTRAN program to evaluate a cosine series up to *n* terms. 4

(c) If A=2·5, B=3·5, J=5 and K=10, what will be the value of J after executing the following program segment? 2

```
      IF (2*K .LE. 3*J) GO TO 50
      J=J+1
      GO TO 60
50    J=K
60    J=J+K
```

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2014

(6th Semester)

PHYSICS

NINTH PAPER

(Method of Mathematical Physics—II)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. Given that $\Gamma(3)\Gamma(\frac{5}{2}) = C\,\Gamma(5)$, the value of C is

(a) $\sqrt{\pi}$ ( )

(b) $\frac{\sqrt{\pi}}{2}$ ( )

(c) $\frac{\sqrt{\pi}}{2^2}$ ( )

(d) $\frac{\sqrt{\pi}}{2^4}$ ( )

2. The value of the ratio $\frac{\Gamma(-\frac{3}{2})}{\Gamma(\frac{3}{2})}$ is

(a) 1 ( )

(b) $\frac{3}{8}$ ( )

(c) $\frac{8}{3}$ ( )

(d) $\frac{2}{3}$ ( )

3. The Fourier series expansion of x^2 in the interval $-\pi \leq x \leq \pi$ is

$$x^2 = \frac{\pi^2}{3} - 4\left\{\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \frac{\cos 4x}{4^2} + \cdots\right\}$$

then the value of $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$ is

(a) $\frac{\pi^2}{3}$ ( )

(b) $\frac{\pi^2}{4}$ ( )

(c) $\frac{\pi^2}{6}$ ( )

(d) $\frac{\pi^2}{12}$ ( )

4. The Fourier sine transform of the function $f(x) = e^{-ax}$ is

(a) $\sqrt{\frac{2}{\pi}} \frac{\omega}{\omega^2 + a^2}$ ( )

(b) $\sqrt{\frac{2}{\pi}} \frac{a}{\omega^2 + a^2}$ ( )

(c) $\sqrt{\frac{2}{\pi}} \frac{\omega \sin ax}{\sqrt{\omega^2 + a^2}}$ ( )

(d) $\sqrt{\frac{2}{\pi}} \frac{a \sin ax}{\sqrt{\omega^2 + a^2}}$ ( )

5. If $f(s)$ is the Laplace transform of $F(t)$, then $\mathscr{L}^{-1}[f(as)]$ is

(a) $\frac{1}{a} F\left(\frac{t}{a}\right)$ ( )

(b) $\frac{1}{a} F\left(\frac{a}{t}\right)$ ( )

(c) $aF\left(\frac{t}{a}\right)$ ( )

(d) $aF\left(\frac{a}{t}\right)$ ( )

6. The Laplace transform of $\delta(t)$ is

(a) 1 ( )

(b) 0 ( )

(c) $\sqrt{2\pi}$ ( )

(d) $\frac{1}{\sqrt{2\pi}}$ ( )

7. In the group $G = \{E, A, A^2\}$, the element conjugate to A^2 is

(a) E ( )

(b) A ( )

(c) A^2 ( )

(d) A^{-2} ( )

8. Two groups $G = \{G_1, G_2, G_3, \ldots, G_n\}$ and $H = \{H_1, H_2, \ldots, H_n\}$ are isomorphic if

(a) $G_1H_1 = G_2H_2$ ( )

(b) $\frac{G_1}{H_1} = \frac{G_2}{H_2}$ ( )

(c) $G_1G_2 = H_1H_2$ ( )

(d) $\frac{G_1}{G_2} = \left(\frac{H_1}{H_2}\right)^2$ ( )

9. The number of times the running variable K will be repeated in the loop DO 10 K=1, 10, 2 is

(a) 2 ( )

(b) 5 ( )

(c) 10 ( )

(d) 3 ( )

10. If $A = 3$, $B = 8$ and $C = 4$, then the value of D in the statement $D = 3 * B / A * C - 4 / J$ is

(a) 12 ( )

(b) 32 ( )

(c) 30 ( )

(d) 22 ( )

SECTION—B

(*Marks* : 15)

Answer the following questions : 3×5=15

1. Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\, d\theta = \frac{\Gamma(\frac{1}{4})\Gamma(\frac{3}{4})}{2}$.

2. Show that FT $[\delta(t)] = \frac{1}{\sqrt{2\pi}}$, where FT denotes Fourier transform

3. Using Laplace transform method, show that the solution of the differential equation $\frac{dx}{dt} + \alpha x = 0$ subject to the initial condition that $x = x_0$ at $t = 0$ is $x = x_0 e^{-\alpha t}$

4. Show that the order of any element of a group is always equal to the order of its inverse.

5. Rewrite the following program segment without using DO loops :

```
      DO 10 K=1, 100
      DO 20 L=1, 25
      WRITE (*, 1) K, L
20    CONTINUE
10    CONTINUE
```

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