2015

(5th Semester)

PHYSICS

SIXTH PAPER

(Quantum Mechanics—I)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. *(a)* What do you mean by duality of radiation and matter? Why does the wave nature of matter not valid for macroscopic particles? Show that the de Broglie wavelength for a material particle of rest mass m_0 and charge q, accelerated from rest through a

potential difference of V volt relativistically is given by

$$\lambda = \frac{h}{\sqrt{2m_0qV\left(1+\frac{qV}{2m_0c^2}\right)}}$$ 1+2+5

(b) What do you mean by the term wave packet? 2

OR

2. *(a)* Write the main features of Bohr's theory of hydrogen atom. Derive the expressions for radius of the orbits and total energy of the electron. 3+2+3

(b) What do you understand by the term complimentarity principle? 2

3. *(a)* What do you mean by Hermitian operator? Show that eigenvalues of Hermitian operators are real. 1+4

(b) What is expectation value? Show that momentum operator is Hermitian. 1+4

OR

4. *(a)* Write down the addition and multiplication conditions to be satisfied by a vector space. 2

(b) Describe Gram-Schmidt orthogonalisation process. Apply the process to find an orthonormal basis set for the subspace U of R^4 spanned by the following vectors : 3+5

$$v_1 = (1, 1, 1, 1)$$
$$v_2 = (1, 2, 4, 5)$$
$$v_3 = (1, -3, -4, -2)$$

5. (a) Obtain Schrödinger time dependent equation. Write the equation in eigenvalue equation form. 6

(b) A wave function is given by $\psi = Ae^{ikx}$, show that the probability current density of the given wave function is given by $J = v|A|^2$, where v is velocity of the particle. 4

OR

6. (a) Show that when the wave packet associated with a particle reduces to a point, it is possible to write $m\frac{d\langle x\rangle}{dt} = \langle p_x\rangle$, where m is mass of the particle, x is position operator, p_x is momentum operator. 8

(b) Write down the conditions to be satisfied by an acceptable wave function in quantum mechanics. 2

7. Obtain the expression for energy eigenvalue of one-dimensional harmonic oscillator. What is zero-point energy of harmonic oscillator? 8+2

OR

8. (a) A free particle of energy E is incident on a potential step given by $V = 0$; $x < 0$ and $V = V_0$; $x \geq 0$. Show that all the waves are reflected when $E < V_0$. 7

(b) Normalised wave function of a free particle in a box is given by $\psi = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$, where $0 < x < L$. Plot the graph for the wave function and the probability density for $n = 2$.

State there are how many antinodes. 3

9. The expression for square of angular momentum is given by

$$L^2 = -\hbar^2 \left\{ \frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial}{\partial\theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\phi^2} \right\}$$

Obtain the eigenvalue of L^2. 10

OR

10. *(a)* Show that square of angular momentum commutes with any one of the components of angular momentum, i.e., $[L^2, L_x] = 0$. What is the physical meaning of the commutation? 4+1

(b) Let $\sigma_x, \sigma_y, \sigma_z$ be Pauli spin matrices. Let $\vec{A}$ and $\vec{B}$ be two vectors. Show that

$$(\vec{\sigma} \cdot \vec{A})(\vec{\sigma} \cdot \vec{B}) = \vec{A} \cdot \vec{B} + i\vec{\sigma} \cdot (\vec{A} \times \vec{B})$$ 5

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2015

(5th Semester)

PHYSICS

SIXTH PAPER

(Quantum Mechanics—I)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 10)

Put a Tick (✓) mark against the correct answer in the brackets provided : 1×10=10

1. de Broglie wavelength of electron accelerated through potential difference of 100 V is about

(a) 12 angstrom ( )

(b) 1·2 angstrom ( )

(c) 0·12 angstrom ( )

(d) 10 angstrom ( )

2. Which of the following is not permissible set of quantum numbers for electrons in an atom?

(a) $n = 4,\ l = 0,\ m = 0,\ s = -1/2$ ( )

(b) $n = 5,\ l = 3,\ m = 0,\ s = +1/2$ ( )

(c) $n = 3,\ l = 2,\ m = -3,\ s = -1/2$ ( )

(d) $n = 3,\ l = 2,\ m = -2,\ s = -1/2$ ( )

3. Let $|\psi\rangle$ be a vector, a be a constant. Then $a|\psi\rangle$ is equal to

(a) $|\psi\rangle a$ ( )

(b) $\langle a\psi|$ ( )

(c) $\langle a\psi|^*$ ( )

(d) $a|\psi\rangle$ ( )

4. If inner product between two vectors is zero, then the two vectors are

(a) perpendicular to each other ( )

(b) parallel to each other ( )

(c) Can be both *(a)* and *(b)* ( )

(d) None of the above ( )

5. Let ψ be a wave function, the quantity $\int \psi^* \psi \, dc$ represents

(a) probability density ( )

(b) probability ( )

(c) energy density ( )

(d) wave intensity ( )

6. According to Ehrenfest theorem, quantum mechanics reduces to classical mechanics when

(a) the wave packet is small ( )

(b) the wave packet is large ( )

(c) the wave packet is of any size ( )

(d) the wave extends to infinity ( )

7. For a free particle in one-dimensional infinite potential, the relation between energy eigenvalue E_n and the quantum state n is given by

(a) $E_n \propto n$ ( )

(b) $E_n \propto n^2$ ( )

(c) $E_n \propto \sqrt{n}$ ( )

(d) $E_n \propto \frac{1}{n^2}$ ( )

8. In quantum tunnelling effect, with the increase in barrier thickness

(a) transmission probability increases ( )

(b) transmission probability decreases ( )

(c) transmission probability first increases then decreases ( )

(d) transmission probability remains the same ( )

9. Eigenvalue of Z component of angular momentum L_Z is given by

(a) $l(l+1)\hbar^2$ ( )

(b) $l(l+1)\hbar$ ( )

(c) $m\hbar$ ( )

(d) $m^2\hbar$ ( )

10. Trace of Pauli spin matrices is

(a) 1 each ( )

(b) $-i$ each ( )

(c) -1 each ( )

(d) 0 each ( )

SECTION—II

(*Marks* : 15)

Give short answers to the following questions : 3×5=15

1. Show that group velocity v_g and phase velocity v_p are related to each other by

$$v_g = v_p - \lambda \frac{dv_p}{d\lambda}$$

2. Let

$$|\psi\rangle = |u_1\rangle + 2|u_2\rangle + i|u_3\rangle \text{ and } |\phi\rangle = 3|u_1\rangle - |u_2\rangle - 2|u_3\rangle$$

Compute the inner product $\langle\psi|\phi\rangle$.

3. Give the physical interpretation of wave function. What does normalization condition mean?

4. Normalized wave function of a free particle in a box is given by $\psi = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$, where $0 < x < L$. Obtain the probability of finding the particle within $0 < x < \frac{2}{L}$.

5. Write a short note on Uhlenbeck and Goudsmit's hypothesis on electron spin.

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