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(5th Semester)

PHYSICS

FIFTH PAPER

(Mathematical Physics—I)

(Revised)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. *(a)* Show that beta function obeys the following identities : 2+2=4

(i) $(m \quad 1, n \quad 1) \quad \frac{n}{m \quad 1} (m, n)$

(ii) $(m \quad 1, n) \quad (m, n \quad 1) \quad (m, n)$

(b) Prove that

$$\int_0^{/2} \sin^p \quad \cos^q \quad d \qquad \frac{\frac{p \quad 1}{2} \quad \frac{q \quad 1}{2}}{2 \quad \frac{p \quad q \quad 2}{2}}$$

Hence show that

$$\int_0^{/2} \sin^3 \quad d \qquad \int_0^{/2} \cos^3 \quad d \qquad \frac{2}{3}$$ 4+2=6

Or

(a) Using the definition of gamma function, find the value of $(\frac{1}{2})$. 4

(b) Show that

(i) $\int_0 e^{\;x^2} dx \quad \frac{\sqrt{\;}}{2}$

(ii) $\int_0^{/2} \cos^{2m \quad 1} \quad \sin^{2n \quad 1} \quad d \qquad \frac{(m, n)}{2}$

3+3=6

2. *(a)* Let $z \quad x \quad iy$ be a complex number. Show that $f(z) \quad z^2$ is analytic function but $f(z) \quad z^{\;1}$ is not an analytic function. 2+2=4

(b) State Cauchy's residue theorem. Use this theorem to show that

$$\oint_C \frac{\sin 2z}{z \quad \overline{4}^{\,3}} dz \qquad 4 \quad i$$

where circle C is defined by $|z| \quad 2$. 1+5=6

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Or

(a) State and prove Cauchy's integral formula. 1+3=4

(b) Use Cauchy's integral formula to evaluate the integrals : 3+3=6

(i) $\circ_C \frac{dz}{z^2 \quad z}$, where C is a circle defined by $|z| \quad 1$

(ii) $\circ_C \frac{2 \quad z}{z(2 \quad z)} dz$, where circle C is $|z| \quad 1$

3. *(a)* Find the inverse of the matrix

$$A \quad \begin{matrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{matrix}$$ 4

(b) Show that the eigenvalues of a Hermitian matrix are real. 3

(c) Show that the matrix

$$\begin{matrix} 0 & i \\ i & 0 \end{matrix}$$

is unitary. Find its eigenvalues. 2+1=3

Or

(a) For a matrix

$$A \quad \begin{matrix} 1 & 2 \\ 3 & 5 \end{matrix}$$

show that $A \ (\text{adj } A) \quad (\text{adj } A) \ A \quad |A| I_n$. 3

(b) Show that any square matrix A can be uniquely expressed as $H_1 \quad iH_2$, where H_1 and H_2 are both Hermitian matrices. What are the expressions for H_1 and H_2? 1+2=3

(c) Solve the following simultaneous equations by matrix method : 4

$$\begin{matrix} 2x & 3y & 4z & 9 \\ & 2y & 3z & 8 \\ & x & z & 3 \end{matrix}$$

4. *(a)* In plane polar coordinates, show that the unit vectors are given by $\hat{r} \quad \hat{i}\cos \quad \hat{j}\sin$ and $\hat{} \quad \hat{i}\sin \quad \hat{j}\cos$. 3

(b) Find the scale factors in orthogonal curvilinear coordinate system. Hence obtain scale factors in Cartesian and cylindrical coordinate systems. 2+2=4

(c) Show that the cylindrical coordinate system is orthogonal. 3

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Or

(a) By writing their transformation relations, explain what you mean by covariant and contravariant vectors. Give one example of each. 2+2=4

(b) Show that Kronecker delta $^{i}_{j}$ is a mixed tensor of rank 2. 3

(c) If a contravariant tensor of rank 2 is symmetric in one coordinate system, show that it is symmetric in any other coordinate system. 3

5. *(a)* With the help of appropriate flowchart diagram, describe how 'while', 'do while' and 'for' loop control statements are executed in C++ programs. 5

(b) Write a C++ program to find the sum of the first N natural numbers and print the result. Use either 'for' loop or 'do while' loop. 3

(c) What are the values of a and b in the following C++ program segment? 2

```
int a, b;
a   5/3   6;
b   6   5/3;
```


Or

(a) Discuss one-dimensional and two-dimensional arrays in C++ with examples. How are character strings arranged in array? 3

(b) Write a C++ program for displaying a (3   3) matrix using an array. Input the elements rowwise and print the matrix in a (3   3) matrix form. 5

(c) What are the values of x and y at the end of the following C++ program segment? 2

```
int x   10, y   20;
x   x   y;
y   x   y;
x   x   y;
```

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Subject Code : PHY/V/05 (R)

Booklet No. A

Date Stamp ....

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DEGREE 5th Semester
(Arts / Science / Commerce /
.......................) Exam., **2017**

Subject ....

Paper ....

To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce /
.......................) Exam., **2017**

Roll No. ....

Regn. No. ....

Subject ....

Paper ....

Descriptive Type
Booklet No. B ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

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Signature of Invigilator(s)

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PHY/V/05 (R)

2 0 1 7

(5th Semester)

PHYSICS

FIFTH PAPER

(Mathematical Physics—I)

(Revised)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 10)

Put a Tick ☑ mark against the correct answer in the boxes provided : 1×10=10

1. The value of $\left(\frac{3}{2}\right)$ is

(a) $\sqrt{\ }$ ☐

(b) $2\sqrt{\ }$ ☐

(c) $\frac{4}{3}\sqrt{\ }$ ☐

(d) $\frac{\sqrt{\ }}{3}$ ☐

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2. The value of (3, 2) is

(a) $\frac{3}{2}$ ☐

(b) $\frac{2}{3}$ ☐

(c) $\frac{1}{10}$ ☐

(d) $\frac{1}{12}$ ☐

3. If $z \quad e^{i}$, then sin is given by

(a) $\frac{1}{2i} \; z \quad \frac{1}{z}$ ☐

(b) $\frac{1}{2} \; z \quad \frac{1}{z}$ ☐

(c) $\frac{1}{2i} \; z \quad \frac{1}{z}$ ☐

(d) $\frac{1}{2} \; z \quad \frac{1}{z}$ ☐

4. The function $f(z) \quad \frac{e^{z}}{z^{2} \quad a^{2}}$ has

(a) two simple poles at $z \quad ia$ and at $z \quad ia$ ☐

(b) two simple poles at $z \quad a$ and at $z \quad a$ ☐

(c) a simple pole at $z \quad a$ and a pole of order 2 at $z \quad a$ ☐

(d) a simple pole at $z \quad ia$ and a pole of order 2 at $z \quad ia$ ☐

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5. Which of the following statements regarding a Hermitian matrix H is true?

(a) e^{iH} is also Hermitian matrix ☐

(b) $H \quad H$, where H is the conjugate of H ☐

(c) The eigenvalues of the matrix H are imaginary ☐

(d) Any two eigenvectors corresponding to two distinct eigenvalues of H are orthogonal ☐

6. The eigenvalues of the matrix

$$A \quad \begin{matrix} 5 & 4 \\ 1 & 2 \end{matrix}$$

are

(a) 5, 4 ☐

(b) 6, 1 ☐

(c) 1, 2 ☐

(d) 4, 1 ☐

7. The scale factors for a spherical polar coordinate system are

(a) $h_1 \quad 1,\ h_2 \quad r,\ h_3 \quad 1$ ☐

(b) $h_1 \quad 1,\ h_2 \quad ,\ h_3 \quad 1$ ☐

(c) $h_1 \quad 1,\ h_2 \quad r,\ h_3 \quad r\sin$ ☐

(d) $h_1 \quad 1,\ h_2 \quad r,\ h_3 \quad \sin$ ☐

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8. If A^i and B_i are contravariant and covariant vectors respectively, then $A^i B_i$ is

(a) invariant tensor ☐

(b) a covariant tensor of rank 2 ☐

(c) a contravariant tensor of rank 2 ☐

(d) a mixed tensor of rank 2 ☐

9. In C++, arithmetic operations $\frac{8}{5}$ and 8%5 respectively result in

(a) 1, 0 ☐

(b) 1, 3 ☐

(c) 3, 1 ☐

(d) 3, 0 ☐

10. What is the final value of y after executing the following three statements in C++?

int x 2, y 3;

x 5;

y y x;

(a) 5 ☐

(b) 7 ☐

(c) 8 ☐

(d) 10 ☐

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SECTION—II

(*Marks* : 15)

Answer the following questions in brief : 3×5=15

1. Show that

$$_0\; x^2 e^{\;\; x^4}\, dx \quad \frac{1}{4} \quad \frac{3}{4}$$

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2. Find the residue of $f(z) \quad \dfrac{2z^2}{4 \quad z^2}$ at all the singularities.

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3. Show that any complex square matrix can be expressed as the sum of a Hermitian matrix and a skew-Hermitian matrix.

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4. The expression of gradient in curvilinear coordinates is

$$\to \qquad \frac{\hat{e}_1}{h_1}\frac{\quad}{u_1} \quad \frac{\hat{e}_2}{h_2}\frac{\quad}{u_2} \quad \frac{\hat{e}_3}{h_3}\frac{\quad}{u_3}$$

Hence write the expressions for gradient in Cartesian, cylindrical and spherical polar coordinates.

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5. Write a C++ program to calculate the area and circumference of a circle. Take radius r as float-type data input and print the output of area (A) and circumference (C) of the circle.

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