2015

(6th Semester)

MATHEMATICS

Paper : Math–363

(Mechanics)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions taking **one** from each Unit

UNIT—I

1. *(a)* Show that a system of coplanar forces acting at different points of a rigid body can be reduced to a single force R through an arbitrary point and a couple G. Further show that the system can be reduced to a single resultant and find the equation of the line of action of the final resultant. 5+2=7

(b) Forces of magnitudes 2, $\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$ act along the lines $x=0$, $y=0$ and $x+y=2$. Find the magnitude of the resultant. 3

2. (a) Two systems of forces P, Q, R and P', Q', R' act along the sides BC, CA, AB of a triangle ABC. Prove that their resultant will be parallel if

$$(QR'-Q'R)\sin A+(RP'-R'P)\sin B+(PQ'-P'Q)\sin C=0$$

5

(b) A beam whose centre of gravity divides it into two portions a and b, is placed inside a smooth sphere. Show that if θ be the inclination to the horizon in the position of equilibrium and 2α be the angle subtended by the beam at the centre of the sphere, then

$$\tan\theta=\frac{b-a}{b+a}\tan\alpha$$

5

UNIT—II

3. (a) A thin uniform wire is bent into the form of a ΔABC and heavy particles of weight P, Q, R are placed at the angular points. Prove that if the centre of mass of the particle coincides with that of the wire, then

$$\frac{P}{b+c}=\frac{Q}{c+a}=\frac{R}{a+b}$$

5

(b) If a piece of wire is bent into a shape of an isosceles triangle whose sides are a and b. Show that the distance of the CG from the base is

$$\frac{a}{2}\sqrt{\frac{2a-b}{2a+b}}$$ 5

4. *(a)* From a circular disc of radius r is cut out a circle described on a radius of the disc as diameter. Find the CG of the remainder. 5

(b) Find the CG of a uniform semicircular disc. 5

UNIT—III

5. *(a)* For a particle moving in a plane curve, show that the radial and transverse acceleration components are given by $f_r = \ddot{r} - r\dot{\theta}^2$ and $f_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$. 5

(b) A point moves in a plane, so that the tangential and normal acceleration are equal and the angular velocity of the tangent is constant and equal to k. Show that the path is equiangular spiral $k_s = Ae^{\psi} + B$, where A and B are constants. 5

6. *(a)* A train travels a distance s in t sec, if it starts from rest and ends at rest. In the first part of the journey it moves with a constant acceleration f and in the second part with a constant retardation f'. Show that if s is the distance between two stations, then

$$t = \sqrt{2s\left(\frac{1}{f} + \frac{1}{f'}\right)}$$ 5

(b) A point execute simple harmonic motion such that in two of its positions the velocities are u, v and the corresponding accelerations are α, β. Show that the distance between the positions is $\frac{v^2 - u^2}{\alpha + \beta}$, and the period and the amplitude of the motion are respectively

$$2\pi\sqrt{\frac{u^2 - v^2}{\beta^2 - \alpha^2}} \text{ and } \frac{\{(v^2 - u^2)(\alpha^2 v^2 - \beta^2 u^2)\}^{\frac{1}{2}}}{(\beta^2 - \alpha^2)}.$$ 5

UNIT—IV

7. *(a)* A particle is projected with initial velocity V from a point O at an angle of elevation α. Show that the range of the plane

through O inclined at an angle β to the horizontal $\left(\beta < \alpha < \frac{\pi}{2}\right)$ is

$$\frac{V^2}{g\cos^2\beta}[\sin(2\alpha - \beta) - \sin\beta]$$

and hence deduce that for a given velocity of propagation V, the maximum range is

$$\frac{V^2}{g(1+\sin\beta)}$$ 5

(b) A particle is projected from a point O at an angle of elevation α and after t sec it has an elevation β as seen from the point of projection. Prove that its initial velocity is

$$\frac{gt\cos\beta}{2\sin(\alpha - \beta)}$$ 5

8. *(a)* A particle is projected vertically upwards with a velocity u against a resistance proportional to the square of the velocity. If V is the terminal velocity of the body and m is its mass, show that, when the body has fallen back to the point of projection, the loss of KE is

$$\frac{1}{2}mu^2\left(\frac{u^2}{V^2+u^2}\right)$$ 5

(b) A body projected at an angle α to the horizon, so as just to clear two walls of equal height a at a distance $2a$ from each other. Show that the range is equal to

$$2a\cot\left(\frac{\alpha}{2}\right)$$ 5

UNIT—V

9. (a) A shot of mass m is fired horizontally from a gun of mass M with a velocity u relative to the gun. Show that the actual velocities of the shot and the gun are

$$\frac{Mu}{M+m} \text{ and } \frac{mu}{M+m}$$

respectively, and that their kinetic energies are inversely proportional to their masses. 5

(b) A ball A of mass m_1 impinges directly on another ball B of mass m_2, which is at rest. After the impact, B impinges directly on a third ball C of mass m_3 which is also at rest. If the velocity imparted to C is the same as A had at first, and if all the balls are perfectly elastic, show that

$$(m_1+m_2)(m_2+m_3)=4m_1m_2$$ 5

10. *(a)* Two balls whose masses are m and m' moving in the direction making angles α and α' with the line of centres, collide. If their direction of motion after the impact be perpendicular to their direction before impact, prove that

$$e = \frac{m\sin^2\alpha' + m'\sin^2\alpha}{m\cos^2\alpha' + m'\cos^2\alpha}$$ 5

(b) A smooth sphere of mass m travelling with velocity u impinges obliquely on a smooth sphere of mass M at rest, the original line of motion of the first sphere making an angle θ with the line of centres at the moment of impact. If the coefficient of restitution be e, show that the impinging sphere will be deflected through a right angle if

$$\tan^2\theta = \frac{eM - m}{M + m}$$

and that its velocity perpendicular to its original line of motion will be obtained if $\theta = 45°$. 5

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2015

(6th Semester)

MATHEMATICS

Paper : Math–363

(Mechanics)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 10)

Put a Tick ☑ mark against the correct answer in the box provided : 1×10=10

1. If a body rests in limiting equilibrium on a rough inclined plane for which the coefficient of friction is $\frac{1}{4}$, then the angle of friction is

(a) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{4}\right)$ ☐

(b) $\pi+\tan^{-1}\left(\frac{1}{4}\right)$ ☐

(c) $\tan^{-1}\left(\frac{1}{4}\right)$ ☐

(d) None of the above ☐

2. Three coplanar forces acting upon a rigid body represented in magnitude, direction and sense by the sides of a triangle taken in order are equivalent to

(a) a single force of magnitude equal to twice the area of the triangle ☐

(b) a couple of moment equal to twice the area of the triangle ☐

(c) a couple of moment equal to the area of the triangle ☐

(d) a single force of magnitude equal to the area of the triangle ☐

3. The CG of a triangle formed by three rods is

(a) the incentre of the triangle ABC ☐

(b) the incentre of the triangle formed by joining the midpoints of the side of triangle ABC ☐

(c) the centroid of the triangle ABC ☐

(d) the orthocentre of the triangle ABC ☐

4. The MI of a uniform thin rod of length $2a$ and of mass M about an axis through the midpoint and perpendicular to the rod is

(a) $\frac{1}{3}Ma^2$ ☐

(b) $\frac{1}{2}Ma^2$ ☐

(c) $\frac{1}{4}Ma^2$ ☐

(d) None of the above ☐

5. For a particle executing SHM of period $\frac{\pi}{10}$ sec and amplitude 5 cm, the maximum velocity attained is

(a) 50 cm/sec ☐

(b) 5 cm/sec ☐

(c) 10 cm/sec ☐

(d) 100 cm/sec ☐

6. If the rate of change of direction of velocity of a particle moving in cycloid is constant, then

(a) tangential acceleration is constant ☐

(b) normal acceleration is constant ☐

(c) resultant acceleration is constant ☐

(d) None of the above ☐

7. If R_1, R_2 are the horizontal ranges of two projectiles projected with the same velocity u from the same point making angles 30° and 60° respectively with the horizontal, then

(a) $R_1 = R_2$ ☐

(b) $R_1 = 2R_2$ ☐

(c) $2R_1 = R_2$ ☐

(d) None of the above ☐

8. The equation of motion of a particle of mass m falling from a point under gravity and resistance equal to $mk(\text{velocity})^2$ is

(a) $\ddot{x} = g + kv^2$ ☐

(b) $\ddot{x} = -g + kv^2$ ☐

(c) $\ddot{x} = -g - kv^2$ ☐

(d) $\ddot{x} = g - kv^2$ ☐

9. A smooth sphere of mass m strikes a plane normally and is rebounded. If e is the coefficient of restitution, then the loss of its KE is

(a) $\frac{1}{2}me^2u^2$ ☐

(b) $\frac{1}{2}mu^2$ ☐

(c) $\frac{1}{2}m(1+e^2)u^2$ ☐

(d) $\frac{1}{2}m(1-e^2)u^2$ ☐

10. A smooth sphere impinges directly with velocity u on another smooth sphere of equal mass at rest. If the spheres are perfectly elastic, the velocity of second sphere after collision will be

(a) u ☐

(b) 0 ☐

(c) $\frac{u}{2}$ ☐

(d) None of the above ☐

SECTION—II

(*Marks* : 15)

Answer the following questions : 3×5=15

1. Find the least force required to pull up a body of mass 50 gm on an inclined plane inclined at an angle 60° to the horizontal.

2. Find the centre of gravity of a segment of a circle.

3. If a particle moves in a plane with constant speed, prove that its acceleration is always perpendicular to its velocity.

4. If a particle is projected with a velocity u from the ground at an angle α with the horizontal, prove that the time to reach the greatest height is $\frac{u \sin \alpha}{g}$

8. A sphere impinges directly on an equal sphere at rest. If the coefficient of restitution be e, show that their velocities after impact are as $(1 - e) : (1 + e)$.

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