VI / MAT (xi)

2014

(6th Semester)

MATHEMATICS

Paper No. : Math–363

(Mechanics)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Forces P, Q, R act along the sides of the triangle formed by the lines $x + y = 1$, $y - x = 1$ and $y = 2$. Find the magnitude of the resultant and the equation of the line along which it acts. 5

(b) If the algebraic sums of the moments of all the coplanar forces acting on a rigid body about three non-collinear points be separately zero, then prove that the body is in equilibrium. 5

2. (a) A uniform ladder is in equilibrium with one end resting on the ground and the other end against a vertical wall; if the ground and wall be both rough, the coefficient of friction being μ and μ' respectively, and the ladder be on the point of slipping at both ends, then show that the inclination of the ladder to the horizon is given by

$$\tan\theta = \frac{1-\mu\mu'}{2\mu}$$ 5

(b) Two rough particles connected by a light string rest on an inclined plane. If their weights and corresponding coefficient of frictions are w_1, w_2 and μ_1, μ_2 respectively, then show that the greatest inclination of the plane for equilibrium is

$$\tan^{-1}\left(\frac{\mu_1 w_1 + \mu_2 w_2}{w_1 + w_2}\right)$$ 5

UNIT—II

3. *(a)* If a triangular lamina ABC hangs at rest with one of the angles A being supported at a fixed point, then prove that the angle which the lower side makes with the horizon is

$$\tan^{-1}\{\frac{1}{2}(\cot B - \cot C)\}.$$ 5

(b) Find the centre of gravity of a uniform arc of a quadrant of the circle $x^2 + y^2 = a^2$ in the positive quadrant. 5

4. *(a)* If a square hole is punched out of a circular lamina, the diagonal of the square being the radius of the circle, then show that the centre of gravity of remainder is at a distance $\frac{a}{8\pi - 4}$ from the centre of the circle, where a is its diameter. 5

(b) State and prove the theorem of parallel axes of the moment of inertia about a rigid body. 5

UNIT—III

5. *(a)* Find out the tangential and normal components of acceleration of a particle moving in a plane curve. 5

(b) A particle rests in equilibrium under the attraction of two centres of force which attract directly as the distance, their intensities being μ and μ'. The particle is displaced slightly towards one of them. Show that the time of a small oscillation is

$$\frac{2\pi}{\sqrt{\mu+\mu'}}$$ 5

6. (a) Two cars start off to race with velocities u and v, and travel in a straight line with uniform accelerations α and β. If the two cars reached the finish line at the same time, then show that the length of the course is

$$\frac{2(u-v)(u\beta-v\alpha)}{(\alpha-\beta)^2}$$ 5

(b) The speed of a train increases at a constant rate α from o to v, then remains constant for an interval and finally decreases to zero at a constant rate β. If l be the total distance described, prove that the total time occupied is

$$\frac{l}{v}+\frac{v}{2}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$$

Also find the least value of time when $\alpha=\beta$. 5

(Continued)

UNIT—IV

7. *(a)* A particle is projected from a point on the ground level and its height is h; when it is at horizontal distance a and $2a$ from its point of projection, prove that the velocity of projection is given by

$$u^2 = \frac{g}{4}\left[\frac{4a^2}{h} + 9h\right]$$ 5

(b) If at any instant the velocity of the projectile be u and its direction of motion α to the horizon, then show that the projectile will move at right angles to this direction after the time $u\,\text{cosec}\,\alpha / g$. 5

8. *(a)* A particle of mass m is falling from a point under gravity in a medium whose resistance is mk (velocity). Find the distance of the particle after time t and show that the terminal velocity is never attained by the particle. 5

(b) A particle of mass m; is projected vertically under gravity; the resistance of the air being mk times the velocity. Show that the greatest height attained by the particle is $\frac{V^2}{g}[\lambda - \log(1+\lambda)]$, where V is the terminal velocity of the particle and λV is its initial vertical velocity. Show that the corresponding time is $\frac{V}{g}\log(1+\lambda)$. 5

UNIT—V

9. *(a)* Deduce work-energy equation. 5

(b) A particle falls from a height h upon a fixed horizontal plane. Show that the whole distance described by the particle before it has finished rebounding is

$$\frac{1+e^2}{1-e^2}h$$

where e is the coefficient of restitution. Also prove that the time that elapses is

$$\sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$$ 5

10. *(a)* Find the loss of kinetic energy in the direct impact of collision of two spheres. 5

(b) A sphere m_1 impinges obliquely on another sphere m_2 which is at rest. If $m_1 = em_2$, then show that they will move at right angles to each other. 5

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2014

(6th Semester)

MATHEMATICS

Paper No. : Math–363

(Mechanics)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. Forces 3, 2, 4, 5 kg (force) act respectively along the sides $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}, \overrightarrow{DA}$ of a square. The magnitude of their resultant is

(a) $\sqrt{8}$ kg (force) ☐

(b) $\sqrt{3}$ kg (force) ☐

(c) $\sqrt{10}$ kg (force) ☐

(d) $\sqrt{12}$ kg (force) ☐

2. If a body on a rough surface is in limiting equilibrium, then

(a) $\mu > \tan\alpha$ ☐

(b) $\mu = \tan\alpha$ ☐

(c) $\mu < \tan\alpha$ ☐

(d) $\mu \leq \tan\alpha$ ☐

3. The centre of gravity of a rod of mass m and its length a is at

(a) $2a$ ☐

(b) $\frac{3}{4}a$ ☐

(c) $\frac{1}{3}a$ ☐

(d) $\frac{1}{2}a$ ☐

4. The moment of inertia of a uniform solid sphere of radius a, mass m about a diameter is

(a) $\frac{1}{5}ma^2$ ☐

(b) $\frac{2}{3}ma^2$ ☐

(c) $\frac{2}{5}ma^2$ ☐

(d) $\frac{1}{24}ma^2$ ☐

5. The frequency of a particle executing SHM with μ as proportionality constant is

(a) $2\pi / \sqrt{\mu}$ ☐

(b) $\pi / \sqrt{\mu}$ ☐

(c) $\mu / 2\pi$ ☐

(d) $\sqrt{\mu} / 2\pi$ ☐

6. If a particle moves in a circle of radius b with $s = b\theta$, then the normal component of acceleration towards the centre is

(a) $b\dot{\theta}^2$ ☐

(b) $b\dot{\theta}$ ☐

(c) $b^2\dot{\theta}^2$ ☐

(d) $b^2\dot{\theta}$ ☐

7. The maximum range of projectile with a velocity u projected from the ground under the gravity is

(a) $2u / g$ ☐

(b) $2u^2 / g$ ☐

(c) u^2 / g ☐

(d) $4u^2 / g$ ☐

8. The terminal velocity of a particle falling under a medium with $\ddot{x} = g - 4k\dot{x}$ as the equation of motion is

(a) g / k ☐

(b) g^2 / k^2 ☐

(c) $2g / k$ ☐

(d) $g / 4k$ ☐

9. If e be the coefficient of restitution of collision of two inelastic bodies, then

(a) $e = 1$ ☐

(b) $e = 0$ ☐

(c) $e = \frac{1}{2}$ ☐

(d) $e = -1$ ☐

10. A smooth sphere of mass m strikes a plane normally and is rebounded. If e be the coefficient of restitution, then the loss of its kinetic energy is

(a) $\frac{1}{2} me^2u^2$ ☐

(b) $\frac{1}{2} m(1 + e^2)u^2$ ☐

(c) $\frac{1}{2} m(1 - e^2)u^2$ ☐

(d) $\frac{1}{2} mu^2$ ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

Write on the following in brief :

1. A uniform ladder rests in equilibrium with its lower end on a rough horizontal plane and its upper end against a smooth vertical wall. If θ be the inclination of the ladder to the vertical, then prove that $\tan\theta = 2\mu$, where μ is the coefficient of friction.

2. Find the centre of mass G of a circular arc of radius a subtending an angle 2α radian at the centre.

3. If the coordinates of a moving point at time t are given by $x = a(2t + \sin 2t)$, $y = a(1 - \cos 2t)$, then show that the acceleration of the point is constant.

4. When a particle falls downward in a resisting medium with resistance varies as the square of the velocity, then prove that the terminal velocity is $\sqrt{g / k}$.

5. When a particle moves under the action of a conservative system of forces, then prove that the sum of its kinetic and potential energies is constant throughout the motion.

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