2017

(6th Semester)

MATHEMATICS

Paper : MATH–362

(Advanced Calculus)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* When do we say that a bounded real function f on $[a, b]$ be Riemann integrable? Show that every continuous function is Riemann integrable. 1+4=5

(b) A function f is bounded and integrable on $[a, b]$ and there exists a function F such that $F' = f$ on $[a, b]$, then prove that $\int_a^b f(x)\,dx = F(b) - F(a)$. 5

2. *(a)* If f_1 and f_2 are two bounded and integrable functions on $[a, b]$, then prove that $f = f_1 + f_2$ is also integrable on $[a, b]$ and $\int_a^b f\,dx = \int_a^b f_1 dx + \int_a^b f_2 dx.$ 5

(b) Compute the value of $\int_{-1}^{1} f\,dx$, where $f(x) = |x|$ by dividing the interval $[-1, 1]$ into $2n$ equal sub-intervals. 5

UNIT—II

3. *(a)* Prove that the improper integral

$$\int_a^b \frac{dx}{(x-a)^n}$$

converges if and only if $n < 1$. 5

(b) Examine the convergence of the following improper integrals : 2½+2½=5

(i) $\int_0^2 \frac{dx}{(1-x)^2}$

(ii) $\int \frac{dx}{(1+x^2)^2}$

4. *(a)* If is continuous in [0,) and

$$\lim_{x \quad 0} (x) \quad {}_0, \ \lim_{x} (x) \quad {}_1$$

then show that

$$_0 \frac{(ax) \qquad (bx)}{x} dx \quad ({}_0 \qquad {}_1) \log \frac{b}{a}$$ 4

(b) Show that

$$^1_0 x^{m \quad 1} (1 \quad x)^{n \quad 1} dx$$

exists if and only if m, n are both positive. 6

UNIT—III

5. *(a)* Prove that uniformly convergent improper integral of a continuous function is itself continuous. 4

(b) If $a \quad b$, then show that

$$_0^{/2} \log \frac{a \quad b \sin}{a \quad b \sin} \frac{d}{\sin} \qquad \sin^{\ 1} \frac{b}{a}$$ 6

6. *(a)* Examine the uniform convergence of the convergent improper integral

$$^1_1 \frac{\cos yx}{\sqrt{1 \quad x^2}} dx$$

in (,). 4

(b) If

$$f(x, y) \quad \frac{y^2}{x^2 \quad y^2} \text{ and } g(y) \quad ^1_0 f(x, y)\, dx$$

then show that the right-hand and left-hand derivatives of g at $y \quad 0$ differ from each other and from $^1_0 f_y(x, 0)\, dx$. 6

UNIT—IV

7. *(a)* Evaluate $x^2 y^2 dx\, dy$ over the region bounded by $x \quad 0, y \quad 0$ and $x^2 \quad y^2 \quad 1$. 4

(b) Show that if $0 \quad h \quad 1$, then

$$^1_h \ ^1_h f(x, y)\, dy \ dx \qquad ^1_h \ ^1_h f(x, y)\, dx \ dy \quad 0$$

but

$$^1_0 \ ^1_0 f(x, y)\, dy \ dx \qquad ^1_0 \ ^1_0 f(x, y)\, dx \ dy$$

where

$$f(x, y) \quad \frac{y^2 \quad x^2}{(y^2 \quad x^2)^2}$$ 6

8. *(a)* Change the order of integration in the integral

$$\int_0^1 dx \int_0^{\sqrt{1-x^2}} \frac{1}{(1+e^y)\sqrt{1-x^2-y^2}}\, dy$$

and hence evaluate it. 5

(b) Show that

$$\int_0^1 \int_0^1 \frac{x-y}{(x+y)^2}\, dy\ dx \neq \int_0^1 \int_0^1 \frac{x-y}{(x+y)^2}\, dx\ dy$$ 5

UNIT—V

9. *(a)* Show that

$$f_n(x) = \frac{n}{x+n}$$

is uniformly convergent on $[0, k]$ whatever k may be but not uniformly convergent in $[0, \infty[$. 5

(b) Examine the term-by-term integration of the series whose sum to first n-terms is $n^2 x(1-x)^n$, $0 \le x \le 1$. 5

10. *(a)* Let $\{f_n\}$ be a sequence of function such that $\lim_{n\to\infty} f_n(x) = f(x)$, $x \in [a, b]$ and let

$$M_n = \sup_{x \in [a, b]} |f_n(x) - f(x)|$$

Then prove that the sequence $\{f_n\}$ converges uniformly to f on $[a, b]$ if and only if $M_n \to 0$ as $n \to \infty$. 5

(b) Examine whether the infinite series

$$\sum_{n=1}^{\infty} \frac{1}{n^3(1+nx^2)}$$

can be differentiated term-by-term between any finite limits. 5

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Subject Code : MATH/VI/10

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 6th Semester
(Arts / Science / Commerce / ....) Exam., **2017**

Subject ....
Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

To be filled in by the Candidate

DEGREE 6th Semester
(Arts / Science / Commerce / ....) Exam., **2017**

Roll No. ....
Regn. No. ....

Subject ....
Paper ....

Descriptive Type
Booklet No. B ....

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

MATH/VI/10

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(6th Semester)

MATHEMATICS

Paper : MATH–362

(Advanced Calculus)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. For any two partitions P_1, P_2 on $[a, b]$ of a bounded function f, we have

(a) $L(P_1, f) \quad U(P_2, f)$ ☐

(b) $L(P_2, f) \quad U(P_1, f)$ ☐

(c) $U(P_2, f) \quad L(P_1, f)$ ☐

(d) $U(P_1, f) \quad U(P_2, f)$ ☐

2. If a bounded function f is integrable on $[a, b]$, then

(a) $\lim\limits_{(P)} \quad S(P, f) \qquad {}_{a}^{b} f\ dx$ ☐

(b) $\lim\limits_{(P)\quad 0} S(P, f) \qquad {}_{a}^{b} f\ dx$ ☐

(c) ${}_{\underline{a}}^{b} f\ dx \qquad {}_{a}^{b} f\ dx$ ☐

(d) $L(P, f) \quad U(P, f) \quad S(P, f)$ ☐

where $L(P, f), U(P, f)$ and $S(P, f)$ are the lower Darboux, upper Darboux and Riemann sum of f corresponding to a partition P of $[a, b]$ with norm $\ (P) \qquad .$

3. Which of the following definite integrals is an improper integral?

(a) ${}_{0}^{/2} \sin x\, dx$ ☐

(b) ${}_{1}^{1} \dfrac{dx}{1 \quad x^2}$ ☐

(c) ${}_{0}^{4} \dfrac{dx}{(x \quad 2)(x \quad 3)}$ ☐

(d) ${}_{0}^{1} \dfrac{dx}{x(1 \quad x)}$ ☐

4. The improper integral $_{0}\ x^{n\ \ 1} e^{\ \ x} dx$ is convergent if and only if

(a) $n \quad 1$ ☐

(b) $n \quad 1$ ☐

(c) $n \quad 1$ ☐

(d) $n \quad 0$ ☐

5. The value of the improper integral

$$_{0}\ e^{\ \ x^2} dx$$

is

(a) $\frac{\sqrt{\ }}{2}$ ☐

(b) $\sqrt{\frac{}{2}}$ ☐

(c) $\frac{}{\sqrt{2}}$ ☐

(d) $\frac{}{2}$ ☐

6. The value of the improper integral

$$_{0}\frac{dx}{a \quad b\cos x}$$

if a is positive and $|b| \quad a$, is

(a) $\dfrac{2}{(a^2 \quad b^2)^{1/2}}$ ☐

(b) $\dfrac{2}{(a^2 \quad b^2)^{3/2}}$ ☐

(c) $\dfrac{}{(a^2 \quad b^2)^{1/2}}$ ☐

(d) $\dfrac{}{(a^2 \quad b^2)^{3/2}}$ ☐

7. The value of the integral

$$_{C}\frac{dx}{x \quad y}$$

where C is the curve $x \quad at^2$, $y \quad 2at$, $0 \quad t \quad 2$ is

(a) $\dfrac{1}{6}$ ☐

(b) $\log 4$ ☐

(c) $\dfrac{1}{6}$ ☐

(d) $\log 2$ ☐

8. The value of the double integral $\frac{x \quad y}{x \quad y} dx\, dy$

over $\frac{1}{2}, 1; \frac{1}{2}, 1$ is

(a) $\frac{1}{2}$ ☐

(b) $\frac{1}{2}$ ☐

(c) $\frac{}{2}$ ☐

(d) 0 ☐

9. With regards to uniform and point-wise convergence of sequences in [a, b], which of the following is true?

(a) Point-wise convergence Uniform convergence ☐

(b) Uniform convergence Point-wise convergence ☐

(c) Uniform limit Point-wise limit ☐

(d) All of the above ☐

10. The sequence of function $f_n(x) \quad nxe^{\quad nx^2}$ is point-wise, but not uniformly convergent on

(a) $[0, k]$, where $k \quad 1$ ☐

(b) $[0, \quad]$ ☐

(c) $[0, \quad)$ ☐

(d) $(0, \quad)$ ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. For the integral $\int_0^1 x\,dx$, find the upper Riemann integral corresponding to the division of [0, 1] into 6 equal interval.

2. Examine the convergence of the improper integral

$$_0 \frac{x^2}{\sqrt{x^5 \quad 1}}\, dx$$

3. Given that

$$_{0}\,\frac{\cos mx}{a^2 \quad x^2}\,dx \qquad \frac{}{2a}\,e^{\quad ma}$$

then prove that

$$_{0}\,\frac{x \sin mx}{1 \quad x^2}\,dx \qquad \frac{}{2}\,e^{\quad m}$$

4. Show that

$$\int_C (x^2 + y^2)\,dy = \frac{46}{15}a^3$$

where C is the arc of the parabola $y^2 = 4ax$ between $(0, 0)$ and $(a, 2a)$.

5. Show that

$$f_n(x) \quad \frac{nx}{1 \quad n^2x^2}$$

is not uniformly convergent in any interval containing zero.

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