2 0 1 6

(6th Semester)

MATHEMATICS

Paper : MATH–362

(Advanced Calculus)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* State and prove Darboux's theorem. 1+4=5

(b) If f is bounded and integrable function on $[a, b]$, then prove that $|f|$ is also integrable on $[a, b]$ and

$$\left| {}_a^b f\,dx \right| \quad {}_a^b |f|\,dx$$ 5

2. *(a)* If a bounded function f is integrable on $[a, c]$ and $[c, b]$, where c is a point of $[a, b]$, then prove that f is also integrable on $[a, b]$. 5

(b) Show that the function $f(x) \quad 3x \quad 1$ is Riemann integrable on $[1, 2]$ and hence show that

$${}_1^2 (3x \quad 1)\,dx \quad \frac{11}{2}$$ 4+1=5

UNIT—II

3. *(a)* Prove that the improper integral

$${}_a^b f\,dx$$

converges at a if and only if to every $\quad 0$ there corresponds $\quad 0$ such that

$$\left| {}_{a \quad 1}^{a \quad 2} f\,dx \right| \quad , 0 \quad {}_1, \quad {}_2$$ 5

(b) Examine the convergence of the following functions : 2½+2½=5

(i) ${}_0^1 \sqrt{\dfrac{1 \quad x}{1 \quad x}}\,dx$

(ii) ${}_0 \dfrac{x \tan^{\ 1} x}{(1 \quad x^4)^{1/3}}\,dx$

4. *(a)* Show that the integral

$$_0\, x^{n \quad 1} e^{\quad x}\, dx$$

is convergent if and only if $n \quad 0$. 5

(b) Prove that every absolutely convergent integral is convergent. 5

UNIT—III

5. *(a)* If $|a| \quad 1$, then show that

$$_0 \log(1 \quad a\cos x)\, dx \qquad \log \frac{1}{2} \quad \frac{1}{2}\sqrt{1 \quad a^2}$$ 5

(b) Let $f(x, y)$ be a continuous function of two variables with rectangle $[a, b; c, d] \quad \mathbb{R}^2$. Then prove that the function defeined by

$$(y) \qquad {}_a^b\, f(x, y)\, dx$$

is continuous in $[c, d]$. 5

6. *(a)* Let f be a real valued continuous function of two variables on the closed rectangle $[a, b; c, d]$. Prove that

$$_c^d \quad a \; _a^b f(x, y)\, dx \;\; dy \qquad _a^b \quad a \; _c^d f(x, y)\, dy \;\; dx$$ 5

(b) Examine the uniform convergence of the convergent improper integral

$$_0\, e^{\quad x^2} \cos yx\, dx \text{ in } (\quad , \quad)$$ 5

UNIT—IV

7. *(a)* Show that

$$_C \frac{y\,dx \quad x\,dy}{x^2 \quad y^2} \qquad 2$$

round the circle $C: x^2 \quad y^2 \quad 1$. 5

(b) Show that

$$_0^1 \; _0^1 \frac{x \quad y}{(x \quad y)^3}\, dy \;\; dx \qquad _0^1 \; _0^1 \frac{x \quad y}{(x \quad y)^3}\, dx \;\; dy$$ 5

8. *(a)* Change the order of integration in the double integral

$$_0^{2a} \; _{\sqrt{2ax \quad x^2}}^{\sqrt{2ax}} f(x, y)\, dy\, dx$$ 5

(b) With the help of Green's theorem, compute the difference between the line integrals

$$I_1 \qquad _{ACB} (x \quad y)^2\, dx \quad (x \quad y)^2\, dy$$

and $$I_2 \qquad _{ADB} (x \quad y)^2\, dx \quad (x \quad y)^2\, dy$$

where ACB and ADB are respectively the straight line and the parabolic arc $y \quad x^2$ joining the points $A(0, 0)$ and $B(1, 1)$. 5

UNIT—V

9. *(a)* If a sequence $\{f_n\}$ converges uniformly to f on x $[a, b]$ and let f_n be integrable n, then prove that f is integrable and

$$\int_a^x f(x)\,dx \quad \lim_{n} \int_a^x f_n(x)\,dx$$ 6

(b) Show that the sequence of function

$$f_n(x) \quad \frac{nx}{e^{nx^2}}$$

is pointwise, but not uniformly convergent on [0, [. 4

10. *(a)* State and prove Cauchy's criterion of uniform convergence of a sequence $\{f_n\}$ of real valued functions on a set E. 6

(b) Examine whether the infinite series

$$_{n \;\; 1} \frac{1}{n^3(1 \quad nx^2)}$$

can be differentiated term by term. 4

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Subject Code : MATH/VI/10

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 6th Semester
(Arts / Science / Commerce / ....) Exam., **2016**

Subject ....

Paper ....

To be filled in by the Candidate

DEGREE 6th Semester
(Arts / Science / Commerce / ....) Exam., **2016**

Roll No. ....

Regn. No. ....

Subject ....

Paper ....

Descriptive Type
Booklet No. B ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

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(6th Semester)

MATHEMATICS

Paper : MATH–362

(Advanced Calculus)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—I

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct alternative in the box provided :

1. The lower Riemann integral for a function f corresponding to the partition P of interval $[a, b]$ is given by the relation

(a) $\sup L(P, f) \quad {}_a^b f\, dx$ ☐

(b) $L(P, f) \quad \sup \; {}_a^b f\, dx$ ☐

(c) $L(P, f) \quad \inf \; {}_a^b f\, dx$ ☐

(d) $\sup U(P, f) \quad {}_a^b f\, dx$ ☐

2. If P and P are two partitions of $[a, b]$ such that P is finer than P, then for a bounded function f

(a) $L(P \; , f) \quad L(P, f)$ ☐

(b) $U(P \; , f) \quad U(P, f)$ ☐

(c) $L(P \; , f) \quad L(P, f)$ ☐

(d) $U(P \; , f) \quad U(P, f)$ ☐

3. If f and g be two positive functions on $[a, b]$ such that

$$\lim_{x \quad a} \frac{f(x)}{g(x)} \quad l$$

a non-zero finite number, then

(a) ${}_a^b g\,dx$ converges if ${}_a^b f\,dx$ converges ☐

(b) ${}_a^b f\,dx$ converges if ${}_a^b g\,dx$ converges ☐

(c) ${}_a^b f\,dx$ diverges if ${}_a^b g\,dx$ diverges ☐

(d) ${}_a^b f\,dx$ and ${}_a^b g\,dx$ behave alike ☐

4. The improper integral ${}_a^b \dfrac{dx}{(x \quad a)^n}$ converges if and only if

(a) $n \quad 1$ ☐ *(b)* $n \quad 1$ ☐

(c) $n \quad 1$ ☐ *(d)* $n \quad 1$ ☐

5. The value of the improper integral ${}_{0}\; e^{\quad x^2} \cos \quad x\,dx$ is

(a) $\dfrac{\sqrt{}}{2}$ ☐ (b) $\dfrac{\sqrt{}}{2} e^{\quad /4}$ ☐

(c) $\dfrac{\sqrt{}}{2} e^{\quad ^2/4}$ ☐ (d) $\dfrac{}{\sqrt{2}} e^{\quad ^2/4}$ ☐

6. The uniformly convergent improper integral of a continuous function

(a) is not continuous ☐

(b) is itself continuous ☐

(c) may be continuous ☐

(d) None of the above ☐

7. The value of the integral ${}_{C}\, xy\,dx$ along the arc of the parabola $x \quad y^2$ from (1, 1) to (1, 1) is

(a) 0 ☐

(b) $\dfrac{2}{5}$ ☐

(c) $\dfrac{4}{5}$ ☐

(d) $\dfrac{5}{4}$ ☐

8. The value of the double integral $x^2y^3\,dx\,dy$ over the circle $x^2 \quad y^2 \quad a^2$ is

(a) 0 ☐

(b) $\frac{1}{2}$ ☐

(c) $\frac{}{2}$ ☐

(d) $\frac{1}{2}$ ☐

9. With regards to uniform and pointwise convergence of sequences in $[a, b]$, which of the following is true?

(a) Pointwise convergence Uniform convergence ☐

(b) Uniform convergence Pointwise convergence ☐

(c) Uniform limit = Pointwise limit ☐

(d) All of the above ☐

10. The sequence $f_n(x) \quad \frac{n}{x \quad n}$ is

(a) uniformly convergent in $[0, k]$, whatever k may be ☐

(b) only pointwise convergent in $[0, k]$, whatever k may be ☐

(c) not uniformly convergent in $[0, k]$, whatever k may be ☐

(d) uniformly convergent in $[0, \quad)$ ☐

Section—II

(*Marks* : 15)

Each question carries 3 marks

1. For any two partitions P_1, P_2 of a bounded function f, show that $L(P_1,\ f)\quad U(P_2,\ f)$.

2. Using Frullani's integral, evaluate

$$_{0} \frac{e^{\quad ax} \qquad e^{\quad bx}}{x}\, dx$$

3. State Weierstrass' M-test for the uniform convergence of a convergent improper integral.

4. Evaluate $_{C}(x^2 \quad y^2)\,dx$, where C is the arc of the parabola $y^2 \quad 4ax$ between (0, 0) and $(a,\ 2a)$.

5. Show that $f_n(x) \quad \dfrac{nx}{1 \quad n^2x^2}$ is not uniformly convergent in any interval containing zero.

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