2014

(6th Semester)

MATHEMATICS

Paper : Math–362

(Advanced Calculus)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Show that a bounded function is Riemann integrable on $[a, b]$ iff for every $\varepsilon > 0$, there exists a partition P of $[a, b]$ such that

$$U(P, f) - L(P, f) < \varepsilon$$ 5

(b) Prove that every continuous function is Riemann integrable. 5

2. *(a)* If f be a continuous function on $[a, b]$ and let $F(x) = \int_a^x f(t)\,dt \quad \forall x \in [a, b]$, then show that

$$F'(x) = f(x) \quad \forall x \in [a, b]$$
5

(b) Give an example of a bounded function which is not R-integrable over the interval [0, 1]. 5

UNIT—II

3. *(a)* Examine the convergence of

$$\int_0^2 \frac{dx}{2x - x^2}$$
5

(b) Examine the convergence of

$$\int_0^\infty x^{n-1} \cdot e^{-x}\,dx$$
5

4. Discuss the convergence of beta function. 10

UNIT—III

5. *(a)* Show that

$$\int_0^{\pi/2} \log(1 - x^2 \cos^2 \theta)\,d\theta = \pi \log[1 + \sqrt{1 - x^2} - \pi \log 2]$$

if $x^2 \le 1$. 5

(b) Define uniform convergence of improper integrals in an infinite range. Show that

$$\int_0^{\infty} e^{-x^2} \cdot \cos yx \, dx$$

is uniform convergent in the interval $]-\infty, \infty[$. 5

6. *(a)* Show that uniform convergent improper integral of a continuous function is itself a continuous function. 5

(b) If f is continuous in $[a, b; c, d]$, then show that

$$\int_a^b \left\{ \int_c^d f(x, y)\, dy \right\} dx = \int_c^d \left\{ \int_a^b f(x, y)\, dx \right\} dy$$ 5

UNIT—IV

7. *(a)* Evaluate $\iint e^{2x+3y} dx\, dy$ over the integral bounded by $x = 0$, $y = 0$ and $x + y = 1$. 5

(b) Evaluate $\iint \frac{xy}{\sqrt{1-y^2}} dx\, dy$ over the positive quadrant of the circle $x^2 + y^2 = 1$. 5

8. *(a)* Find the value of the integral

$$\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dx\, dy$$

by changing the order of integration. 5

(b) Change the order of integration and evaluate

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\,dx\,dy}{\sqrt{x^2+y^2}}$$ 5

UNIT—V

9. State and prove the necessary and sufficient condition for uniform convergence of a sequence. 10

10. (a) Show that the sequence $\left\langle \frac{n}{x+n} \right\rangle$ is uniformly convergent in $[0, k]$ whatever k may be, but not uniformly convergent in $[0, \infty[$. 5

(b) Examine term by term integration for the series for which

$$f_n(x) = n^2 x(1-x)^n, \quad x \in [0, 1]$$ 5

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2014

(6th Semester)

MATHEMATICS

Paper : Math–362

(Advanced Calculus)

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Answer **all** questions

Put a Tick ☑ mark against the correct alternative in the box provided :

1. If P_1 and P_2 are two partitions of the interval $[a, b]$ and $P_1 \subset P_2$, then

(a) $U(P_1, f) \leq U(P_2, f)$ ☐

(b) $U(P_2, f) \leq U(P_1, f)$ ☐

(c) $L(P_2, f) \leq L(P_1, f)$ ☐

(d) $U(P_2, f) \leq L(P_1, f)$ ☐

2. If $f \in R[a, b]$, then

(a) $f^2 \in R[a, b]$ ☐

(b) $f^2 \notin R[a, b]$ ☐

(c) $f^2 \in R(a, b)$ ☐

(d) $f^2 \in R[a, b[$ ☐

3. The value of $\int_0^\infty \frac{dx}{1-x^2}$ is

(a) π ☐

(b) not exist ☐

(c) $\pi / 2$ ☐

(d) ∞ ☐

4. The value of integral $\int_1^\infty \frac{dx}{x(x+1)}$ is

(a) e^2 ☐

(b) $\log 2$ ☐

(c) not exist ☐

(d) $-\log 2$ ☐

5. The value of integral $\int_0^\infty e^{-x^2} dx$ is

(a) $\frac{\pi}{2}$ ☐

(b) $\sqrt{\frac{\pi}{2}}$ ☐

(c) $\frac{\pi}{\sqrt{2}}$ ☐

(d) $\frac{\sqrt{\pi}}{2}$ ☐

6. The value of $\iint xy(x+y)\,dx\,dy$ over the area between $y = x^2$ and $y = x$ is

(a) 56 ☐

(b) $\frac{3}{16}$ ☐

(c) $\frac{3}{46}$ ☐

(d) $\frac{3}{56}$ ☐

7. The value of $\int_0^1 \int_0^1 \frac{x-y}{(x+y)^3} dx\,dy$ is

(a) $\frac{1}{2}$ ☐

(b) $-\frac{1}{2}$ ☐

(c) 2 ☐

(d) -2 ☐

8. In respect of the given series

$$\sum \frac{1}{1+nx}$$

which of the following is correct?

(a) It is uniformly convergent in [0, 1] ☐

(b) 0 is the point of non-uniform convergence of the series ☐

(c) 1 is the point of non-uniform convergence of the series ☐

(d) Cannot be integrable term by term ☐

9. If P_1 and P_2 are any two partitions of a closed and bounded interval $[a, b]$, then the common refinement of P_1 and P_2 is

(a) $P_1 \cap P_2$ ☐

(b) $P_1 \cup P_2$ ☐

(c) $P_1 - P_2$ ☐

(d) None of the above ☐

10. The definite integral $\int_0^2 \frac{dx}{(x-1)(x-2)}$ is

(a) improper integral of first kind ☐

(b) improper integral of second kind ☐

(c) convergent at the left end ☐

(d) convergent at both the end ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. Compute $U(P, f)$ and $L(P, f)$ for the function $f(x) = x$, $0 \le x \le 1$ on taking the partition

$$P = \left\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right\} \text{ of } [0, \ 1]$$

2. Examine the convergence of

$$\int_0^2 \frac{dx}{x(2-x)}$$

3. Prove that the function $\phi(y) = \int_a^b f(x, y)\, dx$ is continuous in $[c, d]$, where $f(x, y)$ is a continuous function of two variables with rectangle $[a, b; c, d] \subset \mathbf{R}^2$.

4. State Green's theorem for double and line integrals

5. Evaluate $\iint xy\,dx\,dy$ over the region in the positive quadrant for which $x + y \leq 1$.

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