GOVERNMENT ZIRTIRI RESIDENTIAL SCIENCE COLLEGE

Subject : Mathematics
Paper Name : Vector calculus and Solid Geometry
Paper No: IV
Semester : IV Semester

A. **Multiple choice questions: [25(5 from each unit)]**
B.

1. If $\hat{a}$ and $\hat{b}$ are two mutually perpendicular proper vectors, then $\hat{a} \times (\hat{b} \times \hat{a})$ is parallel to
   a) $\hat{a}$
   b) $\hat{b}$
   c) $\hat{a} \times \hat{b}$
   d) None of the above

2. The unit tangent vector to the space curve $\vec{r} = t\hat{\imath} + t^2\hat{\jmath} + t^3\hat{k}$ at t=0 is
   a) $\hat{\imath}$
   b) $\hat{\jmath}$
   c) $\hat{k}$
   d) None of the above

3. The value of $[\hat{\imath}\hat{k}\hat{\jmath}]$ is
   a) 1
   b) -1
   c) 0
   d) None of the above

4. The projection of $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ is
   a) $\frac{5}{6}\left(2\hat{\imath} - \hat{\jmath} + \hat{k}\right)$
   b) $\frac{6}{5}\left(2\hat{\imath} - \hat{\jmath} + \hat{k}\right)$
   c) $\frac{6}{5}\left(\hat{\imath} - 2\hat{\jmath} + \hat{k}\right)$
   d) $\frac{5}{6}\left(\hat{\imath} - 2\hat{\jmath} + \hat{k}\right)$

5. If $\hat{a}$ and $\hat{b}$ are non-zero vectors and $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then $\vec{a}$ and $\vec{b}$ are
   a) perpendicular to each other
   b) parallel to each other
   c) neither parallel nor perpendicular
   d) None of the above

6. If the vector $\vec{V} = y^2z\hat{\imath} + a\,x\,y\,z\,\hat{\jmath} + x\,y^2\,\hat{k}$ be conservative vector, then *a* is equal to
   a) 0
   b) 2
   c) 1
   d) None of the above

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7. A vector *f* is said to be irrotational if
   a) $div\,\vec{f} = 0$
   b) $curl\,\vec{f} = 0$
   c) $grad\,(div\,\vec{f}) = 0$
   d) $curl\,(curl\,\vec{f}) = 0$

8. If $\vec{f} = (ax + 3y + 4z)\hat{i} + (x - 2y + 3z)\hat{j} + (3x + 2y - z)\hat{k}$ is solenoidal, then the value of *a* is
   a) 5
   b) 0
   c) 2
   d) 3

9. If $\vec{a}$ is a constant function, then $\vec{a}$ is
   a) both solenoidal and irrotational
   b) solenoidal
   c) irrotational
   d) neither solenoidal nor irrotational

10. If $\vec{a}$ is any vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then $(\vec{a}\,.\nabla)\vec{r}$ is
   a) $\vec{a}$
   b) $\vec{r}$
   c) $\vec{a} \times \vec{r}$
   d) None of the above

11. Which of the following is correct for $3x^2 + 4xy + 5y^2 + 6x + 4y + 7 = 0$?
   (a) $e = 1$
   (b) $e<1$
   (c) $e>1$
   (d) none of these

12. The pair of separate straight lines represented by $6x^2 + 5xy - 4y^2 + 7x + 13y - 3 = 0$ is :
   (a) $2x - y + 3 = 0, 3x + 4y + 1 = 0$
   (b) $2x - y - 3 = 0, 3x + 4y - 1 = 0$
   (c) $2x - y + 3 = 0, 3x + 4y - 1 = 0$
   (d) $2x - y - 3 = 0, 3x + 4y + 1 = 0$

13. The chord of contact of the conic $7x^2 - 8xy + 5y^2 - 4X - 6Y + 5 = 0$ with respect to (-1 , 2) is :
   (a) $17x - 11y - 1 = 0$
   (b) $17x + 11y + 1 = 0$
   (c) $17x + 11y - 1 = 0$
   (d) None of these

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14. The condition that the pair of lines $Ax^2 + 2Hxy + By^2 = 0$ are conjugate diameters of the ellipse $(x^2/a^2) + (y^2/b^2) = 1$ is :
    (a) $Ab^2 + Ba^2 = 0$
    (b) Aa + Bb = 0
    (c) $A^2a + B^2b = 0$
    (d) $Aa^2 + Bb^2 = 0$

15. The equation of directrix of the conic $r\sin^2(\theta/2) = 1$ is :
    (a) $(2/r) = -\cos\theta$
    (b) $(2/r) = \cos\theta$
    (c) $2r = \cos\theta$
    (d) none of these

16. The intercepts made on the X , Y and Z axes by the plane $3x - 4y + 6z - 12 = 0$ are:
    (a) 2,4, -3
    (b) 4, -3, 2
    (c) 3,4,6
    (d) 4,3,6

17. The equation of the plane through the points (0,0,0) , (1,1,0) and (0,1,1) is :
    (a) $x - y - z=0$
    (b) $x + y - z=0$
    (c) $x+ y + z=0$
    (d) $x - y + z=0$

18. The magnitude of the short distance between the line $\frac{x}{4} = \frac{y+1}{3} = \frac{z-2}{2}$ and $5x - 2y - 3z + 6 = 0 = x - 3y + 2z - 3$ is :
    (a) 17/39
    (b) $\sqrt{6}/17$
    (c) $17\sqrt{6}$
    (d) $17\sqrt{6}/39$

19. The equation of the plane through the point (2,3,5) and parallel to the plane $2x - 4y + 3z = 9$ is:
    (a) $2x - 4y + 3z=7$
    (b) $x - y + z=0$
    (c) $3x - 4y + 2z=0$
    (d) none of these

20. The angle of inclination of the line $x + y = 0$, $z= 0$ with z-axis is:
    (a) $\pi/2$
    (b) $\pi/3$
    (c) $\pi/4$
    (d) $\pi/6$

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21. Which of the following coordinate is the end of diameter if the sphere $x^2 + y^2 + z^2 – 6z = 0$ passes through them ?

    (a) (2,0,-2) and (-2,1,2)
    (b) (2,-1,-2) and (1,2,0)
    (c) (2,-2,4) and (-2,2,2)
    (d) none of these

22. Since $f(x,y) = 0$ represent a cylinder when the fixed line is the z-axis and the guiding curve is $f(x,y)=0$ and $z=0$, then which of the following statement is true?

    (a) The cylinder is parallel to z-axis.
    (b) The cylinder is parallel to z-axis.
    (c) The cylinder is perpendicular to z-axis.
    (d) The cylinder is parallel to y-axis.

23. The condition that the plane $lx + my + nz = 0$ touches $ax^2 + by^2 + cz^2 = 0$ is:

    (a) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$
    (b) $\frac{1}{l} + \frac{1}{m} + \frac{1}{n} = 0$
    (c) $\frac{a^2}{m} + \frac{b^2}{l} + \frac{c^2}{n} = 0$
    (d) $\frac{l^2}{a} + \frac{m^2}{b} + \frac{n^2}{c} = 0$

24. The centre and radius of the circle $x^2 + y^2 + z^2\ x + y + z – 4 = 0, x + y + z = 0$ is:

    (a) (0,0,0) & 2
    (b) (1,0,0) & 1
    (c) (0,1,0) & 2
    (d) (1,1,0) & 1

25. The right circular cylinder of radius 4 and axis is the line $x= 2y = -z$ is:

    (a) $x^2 + y^2 + z^2 + 5yz – 3xy + 4xz = 0$
    (b) $5x^2 + 8y^2 + 5z^2 + 4yz + 8zx – 4xy – 144 = 0$
    (c) $5x^2 + 5y^2 + 8z^2 + 4yz – 8zx + xy + 144 = 0$
    (d) none of these

B. Fill in the blanks: (3 from each unit)

1. The vector of magnitude 6 which is perpendicular to both the vectors $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$ is ________

2. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + 6\hat{k}$, then the value of $\vec{a}.(\vec{b} \times \vec{c})$ is ______

3. The value of $(\vec{c} \times \vec{a}) \times (\vec{a} \times \vec{b})$ is ________

4. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{w}$ is a constant vector, then $\vec{w} =$ ______, where $\vec{V} = \vec{w} \times \vec{r}$

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5. If $= x^2 y + 2xyz + z^2$, then *curl grad f* = _____

6. The value of $\nabla . (\nabla \times \vec{F})$ is ____________

7. The common tangent of the circle $x^2 + y^2 = 4ax$ and the parabola $y^2 = 4ax$ is ____.

8. The transformed form of the curve $3x^2 + 4y^2 - 2x - y + 2 = 0$ reffered to the parallel axes through the point (-1,1) is__________________.

9. A set of rectangular axes must be turned without the change of origin so that the expression $7x^2 + 4xy + 3y^2$ will be transformed into the form $ax^2 + by^2$, then the value of a and b are___.

10. The intercepts on x-axis by the plane $x + y + 2z = 2$ is____.

11. The distance of the point (4,3,5) from xz-plane is____.

12. The angle between the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$ and the plane $x + 2y + z - 3 = 0$ is____.

13. If the vertex of the right circular cone is the origin and x-axis is the axis of the cone , and the direction cosine of the axis be (1,0,0). Then the equation of the cone is ____.

14. The angle of intersection of the spheres $x^2 + y^2 + z^2 - 2x - 4y - 6z + 10 = 0$ and $x^2 + y^2 + z^2 - 6x - 2y + 2z + 2 = 0$ is____.

15. The region where a plane cuts a sphere is known as ____.

Answer Key:

A.	**1. (b)**	**2. (a)**	**3. (b)**	**4. (d)**	**5. (a)**
	6. (b)	**7. (b)**	**8. (d)**	**9. (a)**	**10. (a)**
	11.(b)	**12.(c)**	**13.(a)**	**14.(d)**	**15.(a)**
	16.(b)	**17.(d)**	**18.(d)**	**19.(a)**	**20.(c)**
	21.(c)	**22.(b)**	**23.(d)**	**24.(a)**	**25. (b)**

B.	**1.** $(-\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$	**2. 5**	**3.** $[\vec{a}\,\vec{b}\,\vec{c}]\,\vec{a}$
	4. $\frac{1}{2}$ **curl** $\vec{V}$	**5. 0**	**6. 0**
	7. x=0	**8.** $3x'^2 + 4y'^2 - 8x' + 9y' + 10 = 0$	**9. 21 & -11**
	10. 2	**11. 3units**	**12.** $\sin^{-1}\sqrt{\frac{2}{27}}$
	13. $y^2 + z^2 = x^2 \tan^2\theta$	**14.** $\cos^{-1}(2/3)$	**15. Circle**

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