2015

(4th Semester)

MATHEMATICS

Paper : MATH–241

(Vector Calculus and Solid Geometry)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* If ABC be a triangle, then prove that

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$ 3

(b) Find the set of vectors reciprocal to the set $2\hat{i} + 3\hat{j} - \hat{k}$, $\hat{i} - \hat{j} - 2\hat{k}$ and $-\hat{i} + 2\hat{j} + 2\hat{k}$. 4

(c) Find the equation of the tangent to the space curve

$$\vec{r} = t\hat{i} + t^2\hat{j} + \frac{2}{3}t^3\hat{k}$$

at the point $t = 1$. 3

2. (a) Consider the tetrahedron with faces F_1, F_2, F_3, F_4. Let $\vec{V}_1$, $\vec{V}_2$, $\vec{V}_3$ and $\vec{V}_4$ be vectors whose magnitudes are equal to the areas of the faces F_1, F_2, F_3 and F_4 respectively and whose directions are perpendicular to these faces in outward direction. Show that

$$\vec{V}_1 + \vec{V}_2 + \vec{V}_3 + \vec{V}_4 = \vec{0}$$

5

(b) If three concurrent edges of a parallelopiped is given by $\vec{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}$, $\vec{b} = -5\hat{i} + 7\hat{j} - 3\hat{k}$ and $\vec{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$, then find its volume. 5

UNIT—II

3. (a) If $\phi(x, y, z) = x^2 + y^2 + zx$, then find the directional derivative of ϕ in the direction of $\vec{A} = \hat{i} + 2\hat{j} + \hat{k}$ at $(2, -1, 3)$. 4

(b) Prove that

$$\text{curl}\,(\vec{a} \times \vec{b}) = (\vec{b} \cdot \nabla)\vec{a} - \vec{b}(\nabla \cdot \vec{a}) - (\vec{a} \cdot \nabla)\vec{b} + \vec{a}(\nabla \cdot \vec{b})$$

4. (a) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then show that

$$\nabla^2 (|\vec{r}|)^n = n(n+1) |\vec{r}|^{n-2}$$

where n is a constant. 5

(b) Let S be the closed surface enclosing volume V. Using Gauss' divergence theorem, show that

$$\int_V \nabla \phi \, dV = \oint_S \phi \hat{n} \, dS$$ 5

UNIT—III

5. (a) If by rotation of rectangular axes about the origin the expression $ax^2 + 2hxy + by^2$ changes to $a'x'^2 + 2h'x'y' + b'y'^2$, then show that $a + b = a' + b'$ and $ab - h^2 = a'b' - h'^2$. 5

(b) Prove that the chord $y = mx + c$ of the curve subtends a right angle at origin if $c + 4am = 0$. 5

6. (a) Reduce the equation

$$9x^2 - 24xy + 16y^2 - 18x - 101y + 19 = 0$$

to the standard form. 5

(b) Find the equation of the tangent to the conic $4x^2 + 3xy + 2y^2 - 3x + 5y + 7 = 0$ at the point $(1, -2)$. 5

UNIT—IV

7. (a) If a plane cuts the axes at A, B, C and the centroid of the triangle ABC be (a, b, c), then prove that the equation of the plane is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=3$$ 4

(b) Show that the lines $\frac{x+2}{3}=\frac{y-1}{1}=\frac{z+1}{-2}$ and $\frac{x-3}{2}=\frac{y}{-2}=\frac{z+2}{1}$ are coplanar. 4

(c) Find the equation of the plane through the point $(2, 0, -4)$ and parallel to the plane $2x-y+3z=7$. 2

8. (a) Find the equation of the plane which passes through the point $(2, -3, 1)$ and perpendicular to the line joining the points $(4, 5, -2)$ and $(2, -1, 6)$. 4

(b) Find the distance of the point $(1, 2, 3)$ from the line

$$\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-5}{4}$$ 3

(c) Prove that the equation of the plane through the intersection of planes $x+y-2z+4=0$ and $3x-y+2z+1=0$ and parallel to the line

$$\frac{x+2}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$$

is $20x-8y+16z+3=0$. 3

UNIT—V

9. (a) Find the coordinates of the centre of the circle $x^2 + y^2 + z^2 = 30$, $x + 2y + 3z = 14$. 3

(b) Show that the condition for the plane $lx + my + nz = p$ to be a tangent plane to $x^2 + y^2 + z^2 = a^2$ is $a^2(l^2 + m^2 + n^2) = p^2$. 3

(c) Find the radius of the circle where the plane $x - 2y + 2z = 3$ intersects the sphere $x^2 + y^2 + z^2 - 8x + 4y + 8z = 45$. 4

10. (a) The plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

cuts the axes at A, B and C. Find the equation of the cone whose vertex is the origin and the guiding curve is the circle ABC. 5

(b) Find the equation of the cylinder generated by the lines parallel to the line

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$$

and intersecting the guiding curve $z = 3$, $x^2 + y^2 = 4$. 5

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2015

(4th Semester)

MATHEMATICS

Paper : MATH–241

(Vector Calculus and Solid Geometry)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. If $\vec{a}$ and $\vec{b}$ are two mutually perpendicular proper vectors, then $\vec{a}\times(\vec{b}\times\vec{a})$ is parallel to

(a) $\vec{a}$ ☐

(b) $\vec{b}$ ☐

(c) $\vec{a}\times\vec{b}$ ☐

(d) None of the above ☐

2. The unit tangent vector to the space curve $\vec{r} = t\hat{i} + t^2\hat{j} + t^3\hat{k}$ at $t = 0$ is

   (a) $\hat{i}$ ☐

   (b) $\hat{j}$ ☐

   (c) $\hat{k}$ ☐

   (d) None of the above ☐

3. The vector $\vec{V} = -4x\hat{i} + y\hat{j} + cz\hat{k}$ is solenoidal if c is equal to

   (a) 0 ☐

   (b) 1 ☐

   (c) 3 ☐

   (d) 4 ☐

4. Suppose V be the volume bounded by a closed surface S, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\hat{n}$ is the unit vector normal (outward) to the surface S, then $\oint_S \vec{r} \cdot \hat{n}\, dS$ is equal to

   (a) 0 ☐

   (b) V ☐

   (c) $2V$ ☐

   (d) $3V$ ☐

5. The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a parabola, if

(a) $ab - h^2 = 0$ ☐

(b) $ab - h^2 > 0$ ☐

(c) $ab - h^2 < 0$ ☐

(d) None of the above ☐

6. The equation of the normal to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ at (4, 0) is

(a) $x = 0$ ☐

(b) $y = 0$ ☐

(c) $x = 4$ ☐

(d) $y = 3$ ☐

7. The equation of the plane through the points (0, 0, 0), (1, 1, 0) and (0, 1, 1) is

(a) $x - y + z = 0$ ☐

(b) $x + y - z = 0$ ☐

(c) $x + y + z = 0$ ☐

(d) $x - y - z = 0$ ☐

8. The perpendicular distance of the point $(-1, 0, -3)$ from the plane $3x + 4y + 12z = 13$ is

(a) 1 ☐

(b) 4 ☐

(c) 3 ☐

(d) None of the above ☐

9. The equation of the tangent plane to the sphere $x^2 + y^2 + z^2 - x + 3y + 2z - 3 = 0$ at the point $(1, 1, -1)$ is

(a) $x + 5y - z - 3 = 0$ ☐

(b) $5y - 3z - 6 = 0$ ☐

(c) $2x + 3y = 0$ ☐

(d) $x + 5y - 6 = 0$ ☐

10. The equation of the cone whose vertex is the origin and base, the circle $x = a$, $y^2 + z^2 = b^2$ is

(a) $a^2(y^2 + z^2) = b^2x^2$ ☐

(b) $b^2(y^2 + z^2) = a^2x^2$ ☐

(c) $y^2 + z^2 = x^2$ ☐

(d) $y^2 + z^2 = a^2x^2$ ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

State *True* or *False* by putting a Tick ☑ mark in the box provided and give a brief justification :

1. If a space curve given by $x = 3\cos t$, $y = 3\sin t$, $z = 4t$, then the radius of curvature is 3/25.

True ☐ *False* ☐

Justification :

2. The curl of gradient of ϕ, where ϕ is a scalar function of x, y, z, is zero.

True ☐ *False* ☐

Justification :

3. The second degree equation

$$3x^2 + 7xy + 2y^2 - 10x - 5y + 3 = 0$$

represents a pair of straight lines.

True ☐ *False* ☐

Justification :

4. The given straight lines

$$\frac{x-5}{3} = \frac{y-7}{-16} = \frac{z-3}{7} \quad \text{and} \quad \frac{x-9}{3} = \frac{y-13}{8} = \frac{z-15}{-5}$$

are coplanar.

True ☐ *False* ☐

Justification :

5. The line joining the centre of the sphere $x^2 + y^2 + z^2 = a^2$ to any point P is perpendicular to the polar plane of P with respect to the sphere $x^2 + y^2 + z^2 = a$.

True ☐ *False* ☐

Justification :

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