III/MAT (iii)

2016

(3rd Semester)

MATHEMATICS

THIRD PAPER

(Differential Equation)

(Math–231)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Solve the differential equation

$$\tan y \sec^2 x\, dx \quad \tan x \sec^2 y\, dy \quad 0$$ 5

(b) Reduce the equation $x\,dy \quad y\,dx \quad xy^2\,dx$ to exact form and solve it. 5

2. *(a)* Reduce the equation

$$\frac{dy}{dx} \quad \frac{1}{x}\sin 2y \quad x^3 \cos^2 y$$

to a linear differential equation and solve it. 5

(b) Solve the differential equation

$$\frac{dy}{dx} \quad \sin(x \quad y)$$ 5

UNIT—II

3. *(a)* Solve

$$(D^3 \quad D^2 \quad 6D)y \quad 1 \quad x^2$$

where

$$D \quad \frac{d}{dx}$$ 5

(b) Solve

$$(D^2 \quad 2D \quad 1)y \quad x^2 e^{3x}$$

where

$$D \quad \frac{d}{dx}$$ 5

4. *(a)* Solve the differential equation

$$\frac{d^2y}{dx^2} \quad 4y \quad x\cos x$$ 5

(b) Solve

$$(D^2 \quad 6D \quad 9)y \quad 2e^{\quad 3x}$$

where

$$D \quad \frac{d}{dx}$$ 5

UNIT—III

5. *(a)* Solve

$$xyp^2 \quad (x^2 \quad y^2)p \quad xy \quad 0$$

where

$$p \quad \frac{dy}{dx}$$ 5

(b) Find the orthogonal trajectories of the family of curves

$$\frac{x^2}{a^2} \quad \frac{y^2}{b^2 \quad} \quad 1$$

being parameter. 5

6. *(a)* Solve : 5

$$y \quad px \quad p^2 x^4$$

(b) By substituting $x^2 \quad u$ and $y^2 \quad v$, reduce $x^2(y \quad px) \quad yp^2$ into Clairaut's form and find the singular solution. 5

UNIT—IV

7. *(a)* Solve the second-order linear differential equation

$$(x\sin x \quad \cos x)\frac{d^2y}{dx^2} \quad x\cos x\frac{dy}{dx} \quad y\cos x$$
$$\sin x(x\sin x \quad \cos x)^2$$ 5

(b) Solve the homogeneous differential equation

$$x^4\frac{d^3y}{dx^3} \quad 2x^3\frac{d^2y}{dx^2} \quad x^2\frac{dy}{dx} \quad xy \quad 1$$ 5

8. *(a)* Show that the equation

$$(2x^2 \quad 3x)\frac{d^2y}{dx^2} \quad (6x \quad 5)\frac{dy}{dx} \quad 2y \quad (x \quad 1)e^x$$

is exact and solve it. 5

(b) Apply method of variation of parameters to solve the equation

$$(1 \quad x)\frac{d^2y}{dx^2} \quad x\frac{dy}{dx} \quad y \quad (x \quad 1)^2$$ 5

UNIT—V

9. *(a)* Solve the following partial differential equation by Lagrange's method : 5

$$(x^3 \quad 3xy^2)p \quad (y^3 \quad 3x^2y)q \quad 2(x^2 \quad y^2)z$$

(b) Find the equation of surface orthogonal to $\{z(x \quad y)^2, x^2 \quad y^2\} \quad 0$. 5

10. *(a)* Apply Charpit's method to find the complete integral of the equation $px \quad qy \quad pq$. 5

(b) Find the integral surface of $x^2 p \quad y^2 q \quad z^2 \quad 0$ which passes through the hyperbola $xy \quad x \quad y$, $z \quad 1$. 5

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Subject Code : III/ MAT (iii)

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 3rd Semester
(Arts / Science / Commerce /
.......................) Exam., **2016**

Subject ....
Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

To be filled in by the Candidate

DEGREE 3rd Semester
(Arts / Science / Commerce /
.......................) Exam., **2016**

Roll No. ....
Regn. No. ....

Subject ....
Paper ....

Descriptive Type
Booklet No. B ....

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

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(3rd Semester)

MATHEMATICS

THIRD PAPER

(Differential Equation)

(Math–231)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. The differential equation of all circles which passes through the origin and whose centres lie on y-axis is

(a) $(x^2 \quad y^2)\frac{dy}{dx} \quad 2xy \quad 0$ ☐

(b) $(x^2 \quad y^2)\frac{dy}{dx} \quad xy \quad 0$ ☐

(c) $(x^2 \quad y^2)\frac{dy}{dx} \quad 2xy \quad 0$ ☐

(d) $(x^2 \quad y^2)\frac{dy}{dx} \quad xy \quad 0$ ☐

2. Which among the following differential equations is not homogeneous?

(a) $\frac{dy}{dx} \quad \frac{x^2 \quad y^2}{2xy}$ ☐

(b) $(x^2 \quad xy)\frac{dy}{dx} \quad 1$ ☐

(c) $\frac{dy}{dx} \quad \frac{x^2 y}{x^3 \quad y^3}$ ☐

(d) $\frac{dy}{dx} \quad \frac{\sqrt{x^2 \quad y^2} \quad y}{x}$ ☐

3. The particular integral (PI) of the differential equation

$$(D^2 \quad D \quad 1)y \quad e^{\quad x}$$

where $D \quad \frac{d}{dx}$ is

(a) $e^{\quad 2x}$ ☐

(b) e^{x} ☐

(c) e^{2x} ☐

(d) $e^{\quad x}$ ☐

4. While solving linear differential equation with constant coefficient, if the roots of the auxiliary equation has three real and equal roots, say m, then the complementary function will be written as

(a) $(c_1 x \quad c_2 x^2 \quad c_3 x^3) e^{mx}$ ☐

(b) $c_1 e^{mx} \quad c_2 e^{mx} \quad c_3 e^{mx}$ ☐

(c) $(c_1 \quad c_2 x \quad c_3 x^2) e^{mx}$ ☐

(d) $c_1 \quad (c_2 \quad c_3 x) e^{mx}$ ☐

5. The Clairaut's equation of the form $y \quad px \quad f(p)$, where $p \quad \dfrac{dy}{dx}$ has solution, if

(a) $p \quad 0$ ☐

(b) $p \quad c$ ☐

(c) $p \quad y$ ☐

(d) $\dfrac{dp}{dx} \quad 0$ ☐

6. The orthogonal trajectories of a system of concurrent straight line is given by

(a) $x^2 \quad y^2 \quad a^2$ ☐

(b) $2x^2 \quad y^2 \quad a^2$ ☐

(c) $(x \quad 1)^2 \quad y^2 \quad a^2$ ☐

(d) $x^2 \quad y^2 \quad a$ ☐

7. In the linear differential equation of second-order

$$\frac{d^2y}{dx^2} \quad P\frac{dy}{dx} \quad Qy \quad 0$$

where P and Q are functions of x only or constant, if $1 \quad P \quad Q \quad 0$, then y

(a) e^x ☐

(b) $e^{\quad x}$ ☐

(c) x ☐

(d) $\frac{1}{x}$ ☐

8. The differential equation $Pdx \quad Qdy \quad Rdz \quad 0$ is integrable, if

(a) $P \quad \frac{Q}{z} \quad \frac{R}{y} \quad Q \quad \frac{R}{x} \quad \frac{P}{z} \quad R \quad \frac{P}{y} \quad \frac{Q}{x} \quad 0$ □

(b) $P \quad \frac{Q}{x} \quad \frac{R}{y} \quad Q \quad \frac{R}{y} \quad \frac{P}{z} \quad R \quad \frac{P}{z} \quad \frac{Q}{x} \quad 0$ □

(c) $P \quad \frac{P}{y} \quad \frac{Q}{x} \quad Q \quad \frac{R}{x} \quad \frac{P}{z} \quad R \quad \frac{Q}{z} \quad \frac{R}{y} \quad 0$ □

(d) $P \quad \frac{R}{x} \quad \frac{P}{x} \quad Q \quad \frac{P}{y} \quad \frac{R}{x} \quad R \quad \frac{Q}{z} \quad \frac{P}{y} \quad 0$ □

9. The partial differential equation obtained by eliminating arbitrary constants a and b from the equation $z \quad (x^2 \quad a)(y^2 \quad b)$ is

(a) $z \quad pq$ □

(b) $xyz \quad pq$ □

(c) $pq \quad 4xyz$ □

(d) $pq \quad 2xyz$ □

10. The partial differential equation obtained by eliminating a function f from the equation $z \quad xy \quad f(x^2 \quad y^2)$ is

(a) $yp \quad xq \quad y^2 \quad x^2$ ☐

(b) $yp \quad xq \quad y^2 \quad x^2 \quad xy \; \frac{y}{x} \quad \frac{x}{y}$ ☐

(c) $yp \quad xq \quad y^2 \quad x^2 \quad xy \; \frac{y}{x} \quad \frac{x}{y}$ ☐

(d) $yp \quad xq \quad y^2 \quad x^2 \quad xy \; \frac{y}{x} \quad \frac{x}{y}$ ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. Find the integrating factor of the equation

$$y^2 \frac{dy}{dx} \quad x \quad y^3$$

2. Solve

$$(D^2 \quad 4)y \quad x^2$$

where

$$D \quad \frac{d}{dx}$$

3. Solve :

$$p \quad \frac{1}{p} \quad \frac{10}{3}$$

4. Solve the simultaneous linear differential equation

$$\frac{dy}{dx} \quad y \text{ and } \frac{dz}{dx} \quad 2y \quad z$$

5. Form a partial differential equation by eliminating a, b, c from

$$\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad \frac{z^2}{c^2} \quad 1$$

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