2015

(2nd Semester)

MATHEMATICS

SECOND PAPER

(Algebra)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* If A and B are invertible matrices of the same order, then show that AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$. 4

(b) Find two non-singular matrices P and Q such that PAQ is in normal form, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$$

Also find the rank of the matrix A. 6

(*Turn Over*)

2. *(a)* Solve the system of equations : 5

$$x+y+z=6$$
$$x+2y+3z=14$$
$$x-y+z=2$$

(b) Using elementary operation, find the inverse of the matrix

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 3 & 0 \\ 0 & 3 & 4 \end{bmatrix}$$ 5

UNIT—II

3. *(a)* Prove that the inverse of the product of any two elements of a group G is the product of the inverses taken in the reverse order. 3

(b) Define binary operation on a non-empty set A. Verify whether a mathematical system $(\mathbb{Z}, -)$, where '–' is the usual subtraction on $\mathbb{Z}$ is—

(i) commutative;

(ii) associative. 1+1+1=3

(c) Show that every finite group of order less than six must be Abelian. 4

4. (a) Prove that every subgroup of a cyclic group is cyclic. 5

(b) If a and b are any two elements of a group G, then prove that

$$Ha = Hb \Leftrightarrow ab^{-1} \in H$$

where Ha is the right coset of H in G generated by a. 5

UNIT—III

5. (a) State and prove Euler's extension of Fermat's theorem and apply it to show that the remainder on dividing 7^9 by 15 is 7. 6

(b) If f is a homomorphism of a group G into another group $\overline{G}$, prove that $\ker f = \{e\}$ if and only if f is one to one. 4

6. (a) Prove that the order of every element of a finite group is a divisor of the order of the group. 5

(b) If R is the additive group of real numbers and R_+ the multiplicative group of positive real numbers, prove that the mapping $f : R \to R_+$ defined by $f(x) = e^x$ for all $x \in R$ is an isomorphism of R onto R_+. 5

UNIT—IV

7. *(a)* Prove that x^2+x+1 is a factor of $x^{10}+x^5+1$. 4

(b) If a polynomial $f(x)$ is divided by a binomial $(x-a)$, prove that the remainder is $f(a)$. 3

(c) Find the remainder when

$$4x^5+3x^3+6x^2+5$$

is divided by $(2x+1)$. 3

8. *(a)* Find the value of α in order that the expression

$$4x^4-(\alpha-1)x^3+\alpha x^2-6x+1$$

may be divisible by $(2x-1)$. 5

(b) If a polynomial $f(x)$ is divided by

$$(x-a)(x-b),\ a \neq b$$

then prove that the remainder is

$$\frac{(x-b)f(a)-(x-a)f(b)}{a-b}$$ 5

UNIT—V

9. *(a)* Solve by Cardan's method

$$x^3 - 18x - 35 = 0$$ 6

(b) If α, β, γ, δ be the roots of the biquadratic equation

$$x^4 + px^3 + qx^2 + rx + s = 0$$

prove that

$$(\alpha^2 + 1)(\beta^2 + 1)(\gamma^2 + 1)(\delta^2 + 1) = (1 - q - s)^2 + (p - r)^2$$ 4

10. *(a)* If α, β and γ are the roots of the equation

$$x^3 + px^2 + qx + r = 0,$$

form the equation whose roots are

$$\beta^2 + \gamma^2 - \alpha^2,\ \gamma^2 + \alpha^2 - \beta^2,\ \alpha^2 + \beta^2 - \gamma^2$$ 5

(b) The equation $ax^3 + 3bx^2 + 3cx + d = 0$ has two equal roots, prove that

$$(bc - ad)^2 = 4(b^2 - ac)(c^2 - bd)$$ 5

★★★

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(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. If A is an orthogonal matrix, then $A^t A$ is equal to

(a) A^{-1} ☐

(b) $|A|^n$ ☐

(c) I_n ☐

(d) None of the above ☐

2. The rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$ is

(a) 2 ☐

(b) 1 ☐

(c) 0 ☐

(d) 3 ☐

3. The identity element in a group $(\mathbb{Z}, \times)$, where $\mathbb{Z}$ is a set of integers and $\times$ is an ordinary multiplication, is

(a) 0 ☐

(b) 1 ☐

(c) -1 ☐

(d) None of the above ☐

4. The number of binary compositions on a finite set A having n elements is

(a) n^{n^2} ☐

(b) 2^{n^2} ☐

(c) n^n ☐

(d) $n!$ ☐

5. A homomorphism of a group into itself is called

(a) an automorphism ☐

(b) an endomorphism ☐

(c) an isomorophism ☐

(d) None of the above ☐

6. The number of generators of a cyclic group of order 8 is

(a) 2 ☐

(b) 7 ☐

(c) 8 ☐

(d) 4 ☐

7. The polynomial $x^4 - 3x^3 + 2x^2 + 2x - 4$ is divisible by

(a) $x - 1$ ☐

(b) $x - 2$ ☐

(c) $x + 1$ ☐

(d) $x + 2$ ☐

8. Every equation of an even degree whose last term is negative and the coefficient of whose first term is positive has at least two roots,

(a) one positive and one negative ☐

(b) both negative ☐

(c) both positive ☐

(d) None of the above ☐

9. If α, β and γ are the roots of the cubic equation

$$ax^3 + bx^2 + cx + d = 0$$

then $\sum \alpha\beta$ equals

(a) $\frac{d}{a}$ ☐

(b) $\frac{c}{a}$ ☐

(c) $-\frac{b}{a}$ ☐

(d) None of the above ☐

10. The equation $x^4 + 2x^2 + 3x - 1 = 0$ has

(a) two negative roots and two complex roots ☐

(b) two positive roots and two complex roots ☐

(c) one positive, one negative and two complex roots ☐

(d) None of the above ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

State *True* or *False* of the following with a brief justification :

1. If A be a skew-Hermitian matrix, then iA is Hermitian where $i = \sqrt{-1}$.

True ☐ *False* ☐

Justification :

2. The union of two subgroups of a group is always a subgroup

True ☐ *False* ☐

Justification :

3. If $f : G \to G'$ is a homomorphism and $f(G)$ is the homomorphic image of G in G', then $f(G)$ is a subgroup of G'.

True ☐ *False* ☐

Justification :

4. The equation $x^6 + x^4 + x^2 + x + 3 = 0$ has no complex root.

True ☐ *False* ☐

Justification :

8. If $x^4 - 14x^2 + 24x - k = 0$ has four unequal roots, then k must lie between 8 and 11.

True ☐ *False* ☐

Justification :

★ ★ ★