2 0 1 6

(CBCS)

MATHEMATICS

FIRST PAPER

(Calculus)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Draw the graph of the function defined by

$$f(x) \quad \frac{|x|}{[x]}, \; 0 \quad x$$

where $[x]$ is the greatest integer function. Is the function $f(x)$ derivable for all $x \quad (0, \quad)$? 7

(b) Use L'Hospital rule to evaluate

$$\lim_{x \quad 0} \frac{e^x \quad \sin x \quad 1}{\log(1 \quad x)}$$ 3

2. *(a)* Use definition of continuity to prove that $f(x) \quad 2x \quad 5$ is continuous at $x \quad 2$. 4

(b) If $y \quad a\cos(\log x) \quad b\sin(\log x)$, then show that

$$x^2 y_{n \; 2} \quad (2n \quad 1)xy_{n \; 1} \quad (n^2 \quad 1)y_n \quad 0$$ 6

UNIT—II

3. *(a)* State and prove Rolle's theorem. 1+5=6

(b) Expand $\sin x$ in an infinite series in powers of x. 4

4. *(a)* Use Taylor's theorem to express the polynomial $2x^3 \quad 7x^2 \quad x \quad 6$ in powers of $(x \quad 2)$. 5

(b) State and prove Lagrange's mean value theorem. 1+4=5

UNIT—III

5. *(a)* Evaluate : 5

$$\frac{x}{(x \quad 1)(x \quad 2)^2}\,dx$$

(b) Use the definition of the definite integral as a limit of sum to evaluate ${}_1^3 \frac{1}{x}dx$. 5

6. *(a)* Obtain reduction formula for

$$\sin^n x \cos^m x\, dx$$ 6

(b) If

$$I_n \quad {}_0^{\ /2} \sin^n x\, dx$$

show that

$$I_n \quad \frac{n \quad 1}{n} I_{n \quad 2}$$ 4

UNIT—IV

7. *(a)* Let $f : \square^2 \quad \square$ be defined by

$$f(x, y) \quad \begin{matrix} \frac{xy}{x^2 \quad y^2}, & (x, y) \quad (0, 0) \\ 0, & (x, y) \quad (0, 0) \end{matrix}$$

Test the continuity of f at the origin. 5

(b) State and prove Euler's theorem on homogeneous functions. 5

8. *(a)* If

$$u \quad x \quad \frac{y}{x} \qquad \frac{y}{x}$$

show that

$$x^2 \frac{{}^2u}{x^2} \quad 2xy \frac{{}^2u}{x\ y} \quad y^2 \frac{{}^2u}{y^2} \quad 0$$ 5

(b) Find the area enclosed by the circle $x^2 \quad y^2 \quad a^2$. 5

UNIT—V

9. *(a)* Show that the sequence $\{f_n\}$ defined by

$$f_n \quad 1 \quad \frac{1}{2} \quad \frac{1}{3} \quad \cdots \quad \frac{1}{n}$$

cannot converge. 5

(b) Prove that if a sequence is convergent, then it converges to a unique limit. 5

10. *(a)* Test the convergence of the series

$$1 \quad \frac{x}{2} \quad \frac{x^2}{5} \quad \frac{x^3}{10} \quad \cdots \quad \frac{x^n}{(n^2 \quad 1)} \quad \cdots$$ 5

(b) Prove that every convergent sequence is bounded and hence show that

$$\frac{3n \quad 1}{n \quad 1}$$

is bounded above. 3+2=5

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Subject Code :
MATH/I/EC/01 (CBCS)

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

CBCS
DEGREE 1st Semester
(Arts / Science / Commerce /
.......................) Exam., **2016**

Subject ....
Paper ....

To be filled in by the Candidate

CBCS
DEGREE 1st Semester
(Arts / Science / Commerce /
.......................) Exam., **2016**

Roll No. ....
Regn. No. ....

Subject ....
Paper ....

Descriptive Type
Booklet No. B ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

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2 0 1 6

(CBCS)

MATHEMATICS

FIRST PAPER

(Calculus)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. The value of

$$\lim_{x \to 0} \frac{2x}{\sin x}$$

is

(a) –1 ☐

(b) 0 ☐

(c) 1 ☐

(d) 2 ☐

2. The derivative of $\sin x$ with respect to $\cos x$ is

(a) $\quad \tan x$ ☐

(b) $\quad \cot x$ ☐

(c) $\tan x$ ☐

(d) $\cot x$ ☐

3. For which value of $c \quad (1, 5)$, the Rolle's theorem is applicable for the function $f(x) \quad x^2 \quad 6x \quad 5$ in $[1, 5]$?

(a) 1 ☐

(b) 2 ☐

(c) 3 ☐

(d) 4 ☐

4. If the Cauchy's mean value theorem is applicable for the function $f(x) \quad e^x$, $g(x) \quad e^{\ x}$, in $[m, n]$, then the value of $c \quad [m, n]$ is

(a) $\dfrac{m \quad n}{2}$ ☐

(b) $\sqrt{mn}$ ☐

(c) $m \quad n$ ☐

(d) None of the above ☐

5. The value of

$$ {}_{2}^{2}|x|\,dx $$

is

(a) 0 ☐

(b) $\frac{1}{2}$ ☐

(c) 2 ☐

(d) 4 ☐

6. The value of $e^{x}[\sin x \quad \cos x]\,dx$ is

(a) $e^{x}\cos x$ ☐

(b) $e^{x}\sin x$ ☐

(c) $e^{x}\sin x\cos x$ ☐

(d) None of the above ☐

7. Let

$$f(x, y) \quad \frac{3x \quad y}{2x \quad 5y}$$

Then $\lim f(x, y)$ as $(x, y) \quad (\;\, 1, 3)$ along the line $x \quad 1 \quad 0$ is

(a) $\frac{3}{2}$ ☐

(b) 0 ☐

(c) $\frac{1}{5}$ ☐

(d) Does not exist ☐

8. The value of the double integral $\underset{E}{\quad} dydx$ where

$$E \quad (x, y) \quad R^2 : x^2 \quad y^2 \quad \frac{1}{4}$$

is

(a) ☐

(b) $\frac{\quad}{2}$ ☐

(c) $\frac{\quad}{4}$ ☐

(d) None of the above ☐

9. Which one of the following is True regarding sequence?

(a) A bounded sequence is always convergent ☐

(b) If two sequences $\{a_n\}$ and $\{b_n\}$ are both divergent, then $\{a_n \quad b_n\}$ is also divergent ☐

(c) A convergent sequence is bounded ☐

(d) A monotonic sequence is always convergent ☐

10. The series

$$_{n \quad 1} \frac{n!}{n^n}$$

(a) is divergent ☐

(b) is convergent ☐

(c) oscillates finitely ☐

(d) oscillates infinitely ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. *(a)* Using definition of the limit, prove that $\lim_{x \quad 2} x^2 \quad 4.$

Or

(b) If the function

$$f(x) \quad \begin{array}{lll} \dfrac{\sin 4x}{5x} \quad a & \text{for} & x \quad 0 \\ x \quad 4 \quad b & \text{for} & x \quad 0 \\ \quad 1 & \text{for} & x \quad 0 \end{array}$$

is continuous at $x \quad 0$, then find the values of a and b.

2. *(a)* Verify Lagrange's mean value theorem for the function $f(x) \quad 2x^2 \quad 10x \quad 29$ in $[2, 7]$.

Or

(b) Discuss the applicability of Rolle's theorem for the function

$$f(x) \quad \log \frac{x^2 \quad 12}{x}$$

in $[3, 4]$.

3. *(a)* Integrate :

$$e^{x}\ \frac{x \quad 1}{(x \quad 1)^{3}}\ dx$$

Or

(b) Evaluate :

$$\int_{0}^{\;/2} \frac{dx}{1 \quad \sqrt{\cot x}}$$

4. *(a)* Show that for the function

$$f(x, y) = \frac{xy}{x^2 + y^2}$$

the repeated limits

$$\lim_{y \to 0} \lim_{x \to 0} \frac{xy}{x^2 + y^2} \text{ and } \lim_{x \to 0} \lim_{y \to 0} \frac{xy}{x^2 + y^2}$$

exist and are equal. But the double limit

$$\lim_{(x, y) \to (0, 0)} \frac{xy}{x^2 + y^2}$$

does not exist.

Or

(b) If

$$u = \sin^{-1} \frac{x^2 + y^2}{x + y}$$

show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$$

5. *(a)* Show that the sequence $\{b_n\}$, where

$$b_n = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots + \frac{1}{(n+n)^2} = \sum_{k=1}^{n} \frac{1}{(n+k)^2}$$

converges to 0.

Or

(b) Examine the convergence of the series

$$\frac{1}{2 \cdot 4} + \frac{1}{4 \cdot 6} + \frac{1}{6 \cdot 8} + \cdots$$

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