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(6th Semester)

PHYSICS

NINTH PAPER

(Method of Mathematical Physics—II)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(Marks : 50)

The figures in the margin indicate full marks for the questions

1. (a) Using the definition of γ -function, show that

$$\Gamma(m, n) = \frac{(m-1)!(n-1)!}{(m+n-1)!} \quad 4$$

- (b) Evaluate :

$$\int_0^1 \frac{x^2 dx}{\sqrt{(1-x^4)}} + \int_0^1 \frac{dx}{\sqrt{(1-x^4)}} \quad 6$$

Or

- (a) Using the definition of error function, show that

$$\operatorname{erfc}(x) = \operatorname{erfc}(x) \quad 2 \quad 4$$

- (b) Show that

$$\int_0^1 \frac{y^{m-1}}{(1-y)^{m-n}} dy = \int_0^1 \frac{y^{m-1} y^{n-1}}{(1-y)^{m-n}} dy \quad (m, n) \quad 6$$

2. (a) Find the Fourier integral of the function e^{-kx} , where $x > 0$ and $f(x) = f(x), k > 0$.

Hence show that $\int_0^\infty \frac{\cos ux}{u^2+1} du = \frac{1}{2} e^{-x}$ and

also show that $\int_0^\infty \frac{1}{u^2+1} du = \frac{1}{2}$ for $x = 0$.

- (b) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$. 4

Or

- (a) Find the Fourier transform of $e^{-\frac{r^2}{a^2}}$, where a is a constant and $r = \sqrt{x^2 + y^2 + z^2}$. 5

- (b) Find the Fourier integral of the function

$$f(x) = \begin{cases} 0 & ; x < 0 \\ \frac{1}{2} & ; x = 0 \\ e^{-x} & ; x > 0 \end{cases}$$

Hence verify the representation directly at the point $x = 0$. 4+1

(3)

3. (a) Find the Laplace transforms of the functions (i) $e^{at} \cos t$ and (ii) $e^{at} \sin t$. 2+2

(b) Find the inverse Laplace transforms of

$$f(s) = \frac{1}{s(s-2)^3} \quad 6$$

Or

(a) Using Laplace transform, solve the differential equation

$$t \frac{d^2x}{dt^2} - \frac{dx}{dt} - 4tx = 0$$

when $x(0) = 3$ and $x_1(0) = (x)_t=0 = 0$. 6

(b) If $\mathcal{L}[F(t)] = f(s)$, then show that

$$\mathcal{L} \int_0^t F(t) dt = \frac{f(s)}{s}$$

Hence show that

$$\mathcal{L}[t^n F(t)] = (-1)^n \frac{d^n}{ds^n} f(s); \quad n = 1, 2, 3, \dots \quad 2+2$$

4. (a) Prove that the reciprocal of a product of two or more elements of a group is equal to the product of the reciprocals in reverse order. 2

(b) Define inversion centre and explain it in the case of N_2O_2 . 1+3

(4)

(c) Show that the four matrices

$$E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\text{and } C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

form a group under matrix multiplication. 4

Or

(a) Derive the matrix representation for geometric transformations. 6

(b) Show that the set of matrices

$$A = \begin{pmatrix} \cos & \sin \\ \sin & \cos \end{pmatrix}$$

where θ is the real form of a group under multiplication. 4

5. (a) What are the values of I calculated in each one of the following? 1+1

(i) $I = J \cdot 2/3 \cdot K/4 \cdot 6 \cdot J \cdot 3/8$
(where $J = 2$ and $K = 5$)

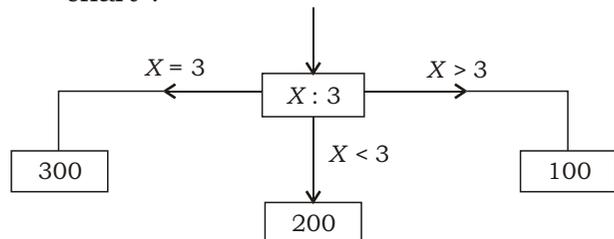
(ii) $I = J/2 \cdot 4 \cdot 3/8 \cdot J \cdot 3$
(where $J = 3$)

(b) Write a FORTRAN program to find the slope and midpoint of a line. 4

(c) Write a DO loop to read 100 numbers and print all numbers less than 30. 2

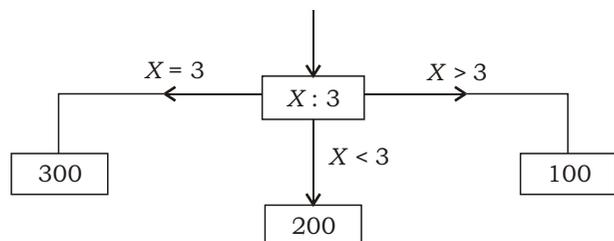
(5)

- (d) Write the FORTRAN program segment using logical IF for the following flow chart : 2



Or

- (a) Write the FORTRAN program segment using IF-THEN-ELSE for the following flow chart : 3



- (b) Given the van der Waals' constants a and b for a gas, write a FORTRAN program to evaluate the critical temperature, pressure and volume using the formula :

$$T_c = \frac{8a}{27Rb}, P_c = \frac{a}{27Rb}, V_c = 3b$$

where $R = 0.0821$.

3

(6)

- (c) Write a FORTRAN DO loop to read the negative numbers between 1 and 100 and print their cubes. 2
- (d) What do you mean by FORMAT specification and explain E format. 2

Subject Code : PHY/VI/09

Booklet No. **A**

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Date Stamp

To be filled in by the Candidate

DEGREE 6th Semester
(Arts / Science / Commerce /
.....) Exam., **2016**
Subject
Paper

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To be filled in by the Candidate

DEGREE 6th Semester
(Arts / Science / Commerce /
.....) Exam., **2016**

Roll No.

Regn. No.

Subject

Paper

Descriptive Type

Booklet No. B

INSTRUCTIONS TO CANDIDATES

- 1. The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
- 2. This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
- 3. While answering the questions of this booklet, any cutting, erasing, overwriting or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

Signature of
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Examiner(s)

Signature of
Invigilator(s)

2 0 1 6

(6th Semester)

PHYSICS

NINTH PAPER

(Method of Mathematical Physics—II)

(PART : A—OBJECTIVE)

(Marks : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(Marks : 10)

Tick (✓) the correct answer in the brackets provided : $1 \times 10 = 10$

1. The value of $\Gamma(m) \Gamma(1 - m)$ is

(a) $\frac{m\pi}{\sin m\pi}$ ()

(b) $\frac{m}{\sin m\pi}$ ()

(c) $\frac{\pi}{\sin m\pi}$ ()

(d) $\frac{\pi}{m \sin m\pi}$ ()

(2)

2. The value of $(1)_n$ is

(a) n ()

(b) $-n$ ()

(c) $n!$ ()

(d) $\frac{1}{n!}$ ()

3. The Fourier transform of $\delta(t)$ is

(a) 1 ()

(b) 0 ()

(c) $\sqrt{2\pi}$ ()

(d) $\frac{1}{\sqrt{2\pi}}$ ()

4. The function $f(x) = x^3 \sin x$ in the range $-\pi < x < \pi$

(a) is an even function ()

(b) is an odd function ()

(c) may be even or odd function ()

(d) is a numeric function ()

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(3)

5. If $f(s)$ is the Laplace transform of $F(t)$, then $\mathcal{L}^{-1}[f(as)]$ is

(a) $\frac{1}{a}F\left(\frac{t}{a}\right)$ ()

(b) $\frac{1}{a}F\left(\frac{a}{t}\right)$ ()

(c) $aF\left(\frac{t}{a}\right)$ ()

(d) $aF\left(\frac{a}{t}\right)$ ()

6. The Laplace transform of $\delta(t)$ is

(a) 1 ()

(b) 0 ()

(c) $\sqrt{2\pi}$ ()

(d) $\frac{1}{\sqrt{2\pi}}$ ()

7. Each irreducible representation of an Abelian group is

(a) one dimensional ()

(b) two dimensional ()

(c) three dimensional ()

(d) n dimensional ($n > 3$) ()

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(4)

8. In the group $G = \{ E, A, A^2 \}$, the element conjugate to A^2 is

(a) E ()

(b) A ()

(c) A^2 ()

(d) A^{-2} ()

9. The final value of I in the DO statement, DO $10I = 1, 10, 2$ is

(a) 1 ()

(b) 10 ()

(c) 2 ()

(d) 9 ()

10. If $I = 3$, $J = 8$ and $K = 4$, then the value of A in the following statement

$$A = 3 * J / I * K - 4 / J$$

is

(a) $\frac{1}{2}$ ()

(b) $\frac{3}{2}$ ()

(c) 1 ()

(d) 0 ()

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(5)

SECTION—II

(Marks : 15)

Answer the following questions :

3×5=15

1. Prove that the factorial function $(\alpha)_n = \frac{\Gamma(\alpha + n)}{\Gamma(\alpha)}$.

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(6)

2. Prove that $\delta(-x) = \delta(x)$.

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(7)

3. Find Laplace transform of t^n , $n > -1$. What will be the Laplace transform of \sqrt{t} ?

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(8)

4. Show that the order of any element of a group is always equal to the order of its inverse.

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(9)

5. Write a small program segment to read 100 numbers and if it is positive, print the square root of it.
