Subject:PHYSICSPaper Name:Thermodynamics & Mathematical PhysicsPaper No.:PHY/II/EC/03Semester:II

A. Multiple Choice Questions:

1. If C be the root mean square velocity of the molecules, and ρ be the density of the gas. Then the pressure P exerted by the gas is

a) $P = \frac{1}{3}\rho^{2}C$ b) $P = \frac{1}{2}\rho^{2}C$ c) $P = \frac{1}{3}\rho C^{2}$ d) $P = \frac{1}{2}\rho C^{2}$

2. For oxygen, the density at *N.T.P.* is $16 \ge 0.000089 \text{ g/cc}$. Then the root mean square velocity is

a) 4.61x10² cm/s b) 4.61x10³ cm/s c) 4.61x10⁴ cm/s d) 4.61x10⁵ cm/s

- 3. At high temperature, a diatomic gas molecule has vibrational degrees of freedom in addition to translational and rotational. Then the ratio $\frac{c_p}{c_n}$ is
 - a) 1.29 b) 1.4 c) 1.33
 - d) 1.67

4. In general gas equation PV=RT, Van der Waal introduced a correction factor a/v² in pressure. This term represent the *a*) effective area of molecules *b*) attractive force between molecules *c*) volume occupied by molecules *d*) mean velocity of gas molecules

5. Which of the following is true for the critical constants?

a) $P_c = \frac{a}{27b^2}$, $V_c = 3b^2$, $T_c = \frac{8a}{27Rb^2}$ b) $P_c = \frac{a^2}{27b}$, $V_c = 3b^2$, $T_c = \frac{8a^2}{27Rb}$ c) $P_c = \frac{a^2}{27b^2}$, $V_c = 3b$, $T_c = \frac{8a^2}{27Rb}$

d)
$$P_c = \frac{a}{27b^2}$$
 , $V_c = 3b$, $T_c = \frac{8a}{27Rb}$

- 6. In Carnot's cycle, the work done during isothermal compression is
 a) -RT₂ log_e V₄/V₃
 b) -RT₂ log_e V₃/V₄
 c) RT₂ log_e V₃/V₄
 - d) $RT_2 \log_e \frac{V_4}{V_3}$
- 7. The physical quantity related to the first law of thermodynamics is
 - a) Energy
 - b) Pressure
 - c) Temperature
 - d) number of moles
- 8. Transfer of heat from a colder body to hotter body is

a) impossibleb) possible by doing some external work

- c) possible when two bodies are kept contact
- d) none of the above
- 9. When 10 grams of ice at 0°C is converted into water at the same temperature (latent heat of ice is 80 cal/gram). The change in entropy is
 a) 293 cal/K
 b) 0.293 cal/K
 c) 29.3 cal/K
 d) 2.93 cal/K
- 10. Which of the following is correct for Maxwell's thermodynamical relation?
 - $a) \left(\frac{\partial V}{\partial S}\right)_{T} = \left(\frac{\partial P}{\partial T}\right)_{V}$ $b) \left(\frac{\partial T}{\partial P}\right)_{S} = \left(\frac{\partial S}{\partial V}\right)_{P}$ $c) \left(\frac{\partial T}{\partial V}\right)_{S} = -\left(\frac{\partial P}{\partial S}\right)_{V}$ $d) \left(\frac{\partial P}{\partial S}\right)_{T} = -\left(\frac{\partial V}{\partial T}\right)_{P}$
- 11. If \vec{F} is a conservative force field, then the value of *curl* \vec{F} is
 - a) 1
 - *b) 0*
 - c) -1
 - $d) \infty$

12. The expression for Gauss divergence theorem is

a)
$$\iiint_{S} div. \vec{F} dS = \iint_{V} \vec{F}. \hat{n} dV$$

b)
$$\iiint_{S} curl \vec{F} dS = \iint_{V} \vec{F}. \hat{n} dV$$

c)
$$\iiint_{V} curl \vec{F} dV = \iint_{S} \vec{F}. \hat{n} dS$$

d)
$$\iiint_{V} div. \vec{F} dV = \iint_{S} \vec{F}. \hat{n} dS$$

13. In curvilinear coordinate system, the gradient of the scalar function ϕ is given by

a)
$$\vec{\nabla}\phi = \sum_{i=1}^{3} \frac{\hat{e}_{i}}{h_{i}} \frac{\partial \phi}{\partial u_{i}}$$

b) $\vec{\nabla}\phi = \sum_{i=1}^{2} \frac{\hat{e}_{i}}{h_{i}} \frac{\partial \phi}{\partial u_{i}}$
c) $\vec{\nabla}\phi = \sum_{i=1}^{3} \frac{h_{i}}{\hat{e}_{i}} \frac{\partial \phi}{\partial u_{i}}$
d) $\vec{\nabla}\phi = \sum_{i=1}^{2} \frac{h_{i}}{\hat{e}_{i}} \frac{\partial \phi}{\partial u_{i}}$

14. The expression for the divergence of a vector field in cylindrical coordinate system is

$$a) \vec{\nabla}.\vec{F} = \frac{1}{r}\frac{\partial}{\partial r}(F_1) + \frac{1}{r^2}\frac{\partial F_2}{\partial \theta} + \frac{\partial F_3}{\partial z}$$

$$b) \vec{\nabla}.\vec{F} = \frac{1}{r}\frac{\partial}{\partial r}(rF_1) + \frac{1}{r^2}\frac{\partial F_2}{\partial \theta} + \frac{\partial F_3}{\partial z}$$

$$c) \vec{\nabla}.\vec{F} = \frac{1}{r}\frac{\partial}{\partial r}(rF_1) + \frac{1}{r}\frac{\partial F_2}{\partial \theta} + \frac{\partial F_3}{\partial z}$$

$$d) \vec{\nabla}.\vec{F} = \frac{1}{r}\frac{\partial}{\partial r}(F_1) + \frac{1}{r}\frac{\partial F_2}{\partial \theta} + \frac{\partial F_3}{\partial z}$$

15. In *n*-dimensional space, the total number of different components that a symmetric tensor of rank two can have is

$$a) \frac{n(2n+1)}{2}$$
$$b) \frac{n(n+1)}{2}$$
$$c) \frac{n(n+2)}{2}$$
$$d) \frac{(n+1)}{2}$$

16. The rank of the matrix $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 2 & 13 & 10 \end{bmatrix}$ is *a)* 2 *b)* 3 *c)* 1 *d)* 4

17. The sum of the eigen values of the matrix $\begin{bmatrix} 3 & 2 & 1 \\ 1 & 3 & 2 \\ 4 & 1 & 5 \end{bmatrix}$ is

- a) 21 b) 10 c) 11
- *d*) 20

18. If
$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$
, then $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & x \end{bmatrix}$. What will be the value of x?
a) 2
b) -2
c) 3
d) -3

19. If A is a nilpotent matrix of order n, then

- a) $A^n = 0$ b) nA = 0c) A = nI, I is identity matrix d) none of the above
- 20. Which of the following property of matrix is correct?

a) multiplication is not commutative

b) multiplication is associative

c) multiplication is distributive over addition

- d) all of the above
- 21. The value of $\beta\left(\frac{1}{2},\frac{1}{2}\right)$ is
 - a) $\sqrt{\pi}$ b) 1
 - c) -1
 - $d) \sqrt{\pi}$
- 22. From the relation between beta and gamma function, which of the following is correct

a)
$$\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

b) $\beta(m, n) = \Gamma(n, m)$
c) $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{(m+n)}$
d) $\beta(m, n) = \frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)}$

23. The value of
$$\int_{0}^{\pi/2} \sqrt{\tan \theta} \, d\theta$$
 is

$$a) \frac{1}{2} \Gamma\left(\frac{3}{4}\right) \Gamma\left(\frac{1}{4}\right)$$

$$b) \frac{3}{2} \Gamma\left(\frac{1}{2}\right) \Gamma\left(\frac{1}{4}\right)$$

$$c) \frac{1}{2} \Gamma\left(\frac{3}{4}\right) \Gamma\left(\frac{5}{4}\right)$$
$$d) \frac{3}{4} \Gamma\left(\frac{3}{4}\right) \Gamma\left(\frac{1}{4}\right)$$

24. From the beta and gamma function, the value of $\int_0^\infty \frac{x^4(1+x^5)}{(1+x)^{15}} dx$ is

a) 5005b) 5500c) $\frac{1}{5005}$ d) $\frac{1}{5500}$

25. Which is not correct for gamma function?

a) $\Gamma(1) = 1$ b) $\Gamma(n + 1) = n!$ c) $\Gamma(n + 1) = n\Gamma(n)$ d) $\Gamma(0) = 1$

B. Fill up the blanks:

- 1. According to the law of equipartition of energy, the average kinetic energy associated with each degree of freedom is ______
- 2. The rate of flow of heat through a metal bar of area of cross-section Im^2 when temperature gradient is I^oC/m under steady state is called _____
- 3. Given that $T_c = 132 K$, $P_c = 37.2 atmos$ and *R per mole* is 82.07 cm³ atmos K⁻¹, the Van der Waals constant b =_____
- 4. The efficiency of Carnot's engine working between steam point and ice point is
- 5. In a cycle of reversible process, the entropy of a system remains ______
- 6. The expression for Clausius-Clapeyron's equation is _____
- 7. The divergence of the vector $\vec{V} = (xyz)\hat{\imath} + (3x^2y)\hat{\jmath} + (xz^2 y^2z)\hat{k}$ at (2, -1, 1) is
- 8. The velocity of a fluid at any point is a contravariant vector of rank

9. Kronecker delta is a mixed tensor of rank _____

10. If <i>H</i> is a <i>Hermitian matrix</i> , then e^{iH} is a matrix.
11. The eigen values of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ are
12. If the order of matrix <i>A</i> is <i>m</i> x <i>p</i> , and the order of <i>B</i> is <i>p</i> x <i>n</i> . Then the order of the matrix <i>AB</i> is
13. The value of the integral $\int_0^{\pi/2} \sin^p \theta \cos^q \theta d\theta$ is
14. The value of the integra $\int_0^{\pi/2} \sin^6\theta \ d\theta$ is
15. The value of $\Gamma\left(\frac{1}{2}\right)$ is
Key Answers
A. Multiple Choice Questions:
1. c) $P = \frac{1}{2}\rho C^2$
2. c) $4.61 \times 10^4 \text{ cm/s}$
3. a) 1.29
4. b) attractive force between molecules
5. d) $P_{c} = \frac{a}{27k^{2}}$, $V_{c} = 3b$, $T_{c} = \frac{8a}{27k^{2}}$
6. b) $-RT_2 \log_e \frac{V_3}{V_4}$
7. a) Energy
8. b) possible by doing some external work
9. d) 2.93 cal/K
10. c) $\left(\frac{\partial T}{\partial V}\right)_{c} = -\left(\frac{\partial P}{\partial S}\right)_{U}$
11. <i>b</i>) 0
12. d) $\iiint_V div. \vec{F} dV = \iint_S \vec{F}. \hat{n} dS$
13. a) $\vec{\nabla}\phi = \sum_{i=1}^{3} \frac{\hat{e}_i}{h_i} \frac{\partial \phi}{\partial u_i}$
14. c) $\vec{\nabla} \cdot \vec{F} = \frac{1}{r} \frac{\partial}{\partial r} (rF_1) + \frac{1}{r} \frac{\partial F_2}{\partial \theta} + \frac{\partial F_3}{\partial z}$
15. b) $\frac{n(n+1)}{2}$
16. <i>a</i>) 2
17. c) 11
18. <i>d</i>) -3
19. <i>a</i>) $A^n = 0$
20. d) all of the above
21. <i>b</i>) <i>1</i>

22. a) $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ 23. a) $\frac{1}{2}\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{1}{4}\right)$ 24. c) $\frac{1}{5005}$ 25. d) $\Gamma(0) = 1$

B. Fill up the blanks:

$1.\frac{1}{2}kT$	2. Thermal conductivity	3. 36.41 cm^3
4. 26.81 %	5. Constant/unchanged	$6. \frac{dP}{dT} = \frac{L}{T(V_2 - V_1)}$
7.14	8. <i>one</i>	9. <i>two</i>
10. unitary	11. <i>1</i> , <i>1</i> , <i>3</i>	12. <i>m</i> x <i>n</i>
13. $\frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}$	$14.\frac{5\pi}{32}$	15. $\sqrt{\pi}$