# 2017

( CBCS )

( 2nd Semester )

## PHYSICS

SECOND PAPER

**( Thermodynamics and Mathematical Physics—I )**

*Full Marks* : 75

*Time* : 3 hours

( PART : B—DESCRIPTIVE )

( *Marks* : 50 )

*The figures in the margin indicate full marks for the questions*

**1.** *(a)* Outline the essential features of the kinetic theory of gases. Derive an expression for the pressure of an ideal gas on the basis of kinetic theory of gases. 2+5=7





*(b)* The critical temperature and pressure of $CO_2$ is 31 °C and 73 atmospheres respectively. Assuming that $CO_2$ obeys van der Waals' gas equation, estimate the diameter of $CO_2$ molecule. 3

*Or*

*(a)* Discuss briefly the considerations that led van der Waals' to modify the gas equation of state. Hence deduce van der Waals' gas equation. 2+5=7

*(b)* Show that for an adiabatic process, $PV$ constant. 3

**2.** *(a)* Explain absolute or thermodynamic scale of temperature. Show that in the thermodynamic scale of temperature, any two temperatures are in the same ratio as the quantities of heat absorbed and rejected by Carnot's reversible engine operating between these two temperatures. 5

*(b)* Entropy of an isolated system remains constant or increases according as it undergoes reversible or irreversible changes. Explain. 3

*(c)* A Carnot engine has the same efficiency between 1000 K and 500 K and between $x$ K and 1000 K (temperature of sink). Calculate $x$. 2

**5.** *(a)* Establish the relation : 5

$$(m, n) \quad \frac{(m) \quad (n)}{(m \quad n)}$$

*(b)* Using the definition of -functions, evaluate the integrals : 5

(i) $\int_0 e^{\ ax} x^{m \ \ 1} \cos bx \ \ dx$

(ii) $\int_0 e^{\ ax} x^{m \ \ 1} \sin bx \ \ dx$

**OR**

*(a)* Using the definition of -functions, prove that $(\tfrac{1}{2}) \quad \sqrt{\ }$. 5

*(b)* Prove that

$$(m, n) \quad {}_0 \frac{x^{m \ \ 1}}{(1 \quad x)^{m \ \ n}} dx$$

Hence show that

$${}_0 \frac{x^8 (1 \quad x^6)}{(1 \quad x)^{24}} \ dx \quad 0$$

3+2=5

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**4.** *(a)* Define the following with one example of each : 3

(i) Hermitian matrix

(ii) Unitary matrix

(iii) Orthogonal matrix

*(b)* Justify that $A$ (adj $A$) (adj $A$) $A$, where

$$A \quad \begin{matrix} 1 & 2 \\ 3 & 5 \end{matrix}$$

3

*(c)* Find the inverse of the matrix

$$\begin{matrix} \cos & \sin \\ \sin & \cos \end{matrix}$$

4

*Or*

*(a)* Find the eigenvalues and eigenvectors of the following matrix : 5

$$\begin{matrix} 0 & i \\ i & 0 \end{matrix}$$

*(b)* Using the matrix method, solve the following algebraic equations : 5

$$x \quad 2y \quad 3z \quad 10$$
$$2x \quad 3y \quad z \quad 1$$
$$3x \quad y \quad 2z \quad 9$$

*Or*

*(a)* Deduce the Maxwell's four thermodynamic relations by using thermodynamic potentials. 6

*(b)* Deduce the Clausius-Clapeyron latent heat equation. 3

*(c)* State the Zeroth law of thermodynamics. 1

**3.** *(a)* If $u \quad 2x \quad 3,\ v \quad y \quad 4,\ w \quad z \quad 2$; show that $u, v, w$ are orthogonal and find $ds^2$ and the metrical coefficients $h_1, h_2, h_3$. 4

*(b)* If is a scalar field and $\vec{A}$, a vector field, find the value of $\vec{\ } \ (\ \vec{A})$. 3

*(c)* Define contravariant tensor. Show that the velocity of a fluid at any point is a contravariant vector of rank one. 1+2=3

*Or*

*(a)* Show that the Kronecker delta is a mixed tensor of rank two. 3

*(b)* Derive an expression for the divergence in terms of orthogonal curvilinear coordinates. 3

*(c)* Prove that

$$\text{curl}(\ \vec{A}) \quad \text{curl}\,\vec{A} \quad \text{grad} \quad \vec{A}$$ 4





**4.** *(a)* Define the following with one example of each : 3

*(i)* Hermitian matrix

*(ii)* Unitary matrix

*(iii)* Orthogonal matrix

*(b)* Justify that $A \ (\text{adj}\ A) \quad (\text{adj}\ A)\ A$, where

$$A \quad \begin{array}{cc} 1 & 2 \\ 3 & 5 \end{array}$$ 3

*(c)* Find the inverse of the matrix

$$\begin{array}{cc} \cos & \sin \\ \sin & \cos \end{array}$$ 4

*Or*

*(a)* Find the eigenvalues and eigenvectors of the following matrix : 5

$$\begin{array}{cc} 0 & i \\ i & 0 \end{array}$$

*(b)* Using the matrix method, solve the following algebraic equations : 5

$$x \quad 2y \quad 3z \quad 10$$
$$2x \quad 3y \quad z \quad 1$$
$$3x \quad y \quad 2z \quad 9$$

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*(a)* Deduce the Maxwell's four thermodynamic relations by using thermodynamic potentials. 6

*(b)* Deduce the Clausius-Clapeyron latent heat equation. 3

*(c)* State the Zeroth law of thermodynamics. 1

**3.** *(a)* If $u \quad 2x \quad 3$, $v \quad y \quad 4$, $w \quad z \quad 2$; show that $u$, $v$, $w$ are orthogonal and find $ds^2$ and the metrical coefficients $h_1, h_2, h_3$. 4

*(b)* If  is a scalar field and $\vec{A}$, a vector field, find the value of $\vec{\ } \ (\ \vec{A})$. 3

*(c)* Define contravariant tensor. Show that the velocity of a fluid at any point is a contravariant vector of rank one. 1+2=3

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*(a)* Show that the Kronecker delta is a mixed tensor of rank two. 3

*(b)* Derive an expression for the divergence in terms of orthogonal curvilinear coordinates. 3

*(c)* Prove that

$$\text{curl}(\ \vec{A}) \quad \text{curl}\,\vec{A} \quad \text{grad} \quad \vec{A}$$ 4

**Subject Code :**
**PHY/II/EC/03 (CBCS)**

**Booklet No. A**

Date Stamp ............................

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**To be filled in by the Candidate**

CBCS
DEGREE 2nd Semester
(Arts / Science / Commerce /
....................... ) Exam., **2017**

Subject .......................................
Paper ..........................................

**To be filled in by the Candidate**

CBCS
DEGREE 2nd Semester
(Arts / Science / Commerce /
....................... ) Exam., **2017**

Roll No. .................................
Regn. No. ..............................

Subject ..................................
Paper .....................................

Descriptive Type
Booklet No. B ........................

**INSTRUCTIONS TO CANDIDATES**

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

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*Signature of Examiner(s)*

*Signature of Invigilator(s)*

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(CBCS)

( 2nd Semester )

# PHYSICS

SECOND PAPER

**( Thermodynamics and Mathematical Physics—I )**

( PART : A—OBJECTIVE )

( *Marks* : 25 )

*The figures in the margin indicate full marks for the questions*

## SECTION—I

( *Marks* : 10 )

Put a Tick (✓) mark against the correct answer in the brackets provided : 1×10=10

**1.** Molecules of an ideal gas have

*(a)* only potential energy ( )

*(b)* only kinetic energy ( )

*(c)* both potential and kinetic energy ( )

*(d)* no energy ( )

**2.** Two gases have equal reduced pressure and reduced temperature, then

*(a)* the gas having larger molecular weight will have larger reduced volume ( )

*(b)* the gas having larger molecular weight will have smaller reduced volume ( )

*(c)* two gases will have same reduced pressure ( )

*(d)* No conclusion can be drawn about reduced volume ( )

**3.** The thermodynamic process which takes place at constant temperature is called

*(a)* isothermal process ( )

*(b)* adiabatic process ( )

*(c)* isobaric process ( )

*(d)* isochoric process ( )

**4.** Net work done by a Carnot's reversible heat engine per cycle is equal to

*(a)* the area included between isothermal at $T_1$ and volume axis ( )

*(b)* the area included between isothermal at $T_2$ and volume axis ( )

*(c)* the area included between two isothermals and two adiabatics ( )

*(d)* zero ( )

**5.** If three surfaces $u = \text{constant}$, $v = \text{constant}$ and $w = \text{constant}$ intersect at a point $P$, then the values of $u$, $v$ and $w$ at $P$ are called

*(a)* coordinate surfaces of the point $P$ ( )

*(b)* curvilinear coordiantes of the point $P$ ( )

*(c)* coordinate curves of the point $P$ ( )

*(d)* loci of the point $P$ ( )

**6.** If $A_{\mu\nu}$ is a skew-symmetric tensor of second order and $B^{\mu}$ is a tensor of rank one, then $A_{\mu\nu}B^{\mu}B^{\nu}$ is equal to

*(a)* $\infty$ ( )

*(b)* $-1$ ( )

*(c)* 0 ( )

*(d)* 1 ( )

**7.** The determinant of a square matrix $A$ is 10, then the determinant of the transpose of $A$ is

*(a)* $-10$ ( )

*(b)* 10 ( )

*(c)* $\frac{1}{10}$ ( )

*(d)* $-\frac{1}{10}$ ( )

**8.** If $A^{\Theta}$ be the conjugate transpose of a matrix $A$, then $(A^{\Theta})^{\Theta}$ is equal to

*(a)* $A - A^{\Theta}$ ( )

*(b)* $A + A^{\Theta}$ ( )

*(c)* $A$ ( )

*(d)* $-A$ ( )

**9.** Given that $\Gamma(3)\Gamma\left(\frac{5}{2}\right) = C\,\Gamma(5)$, then the value of $C$ is

*(a)* $\sqrt{\pi}$ ( )

*(b)* $\frac{\sqrt{\pi}}{2}$ ( )

*(c)* $\frac{\sqrt{\pi}}{2^2}$ ( )

*(d)* $\frac{\sqrt{\pi}}{2^4}$ ( )

**10.** The value of $\Gamma(1+m)\Gamma(1-m)$ is

(a) $\dfrac{m\pi}{\sin m\pi}$ ( )

(b) $\dfrac{\pi}{\sin m\pi}$ ( )

(c) $\dfrac{\pi}{m \sin m\pi}$ ( )

(d) $\dfrac{\sin(1-m)\pi}{\sin(1+m)\pi}$ ( )

## SECTION—II

( *Marks* : 15 )

Answer any *five* of the following questions : 3×5=15

**1.** What do you mean by law of corresponding states?

**2.** Using the kinetic theory of gases, derive the perfect gas equation.

**3.** Derive an expression for entropy of one mole of a perfect gas in terms of temperature $T$ and volume $V$.

**4.** State and explain first law of thermodynamics.

**5.** If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that $\vec{\nabla} r = \hat{r}$.

**6.** Show that curl grad $\phi$ of a scalar function $\phi$ is always zero.

**7.** Show that every square matrix can uniquely be expressed as the sum of a symmetric matrix and a skew-symmetric matrix.

**8.** Show that any matrix $A$ can be uniquely expressed as $H_1 + iH_2$, where $H_1$ and $H_2$ are both Hermitian.

**9.** Using the definition of $\Gamma$-function, prove that $\Gamma(n+1) = n\Gamma(n)$.

**10.** Using the definition of $\beta$-function, show that $\beta(m, n) = \beta(n, m)$.

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