V/PHY (vi) (R)

2 0 1 6

(5th Semester)

PHYSICS

SIXTH PAPER

(Quantum Mechanics—II)

(Revised)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. What is de Broglie hypothesis? Describe Davisson-Germer experiment for the study of electron diffraction. What are the results of the experiment? 2+8=10

Or

(a) Obtain Schrödinger time-independent equation. Write the equation in eigenvalue equation form. 6

(b) Write four basic postulates of quantum mechanics. 4

2. A beam of particles of mass m and energy E is incident from the left on a rectangular potential barrier of the form

$$V(x) \quad \begin{array}{l} 0\ , \quad x \quad 0 \\ V_0\ , \quad 0 \quad x \quad a \\ 0\ , \quad x \quad a \end{array}$$

where V_0 is the height and a is the thickness of the potential barrier. Discuss the solution for $E \quad V_0$ and explain how tunnelling can be understood without violation of energy. Give two examples of quantum tunnelling. 9+1=10

Or

(a) What do you mean by operator in quantum mechanics? What is a linear operator? The operator $x \dfrac{d}{dx}$ has the eigenvalue . Obtain its eigenfunction. 1+1+2=4

(b) What do you mean by eigenvalues and eigenvectors of an operator? For a Hermitian operator, show that the eigenvectors corresponding to different eigenvalues are orthogonal. 2+4=6

3. What is zero point energy of harmonic oscillator? Derive the expression for the eigenfunction in terms of Hermite polynomials of a linear harmonic oscillator. 2+8=10

Or

Solve the radial equation

$$\frac{1}{r^2}\,\frac{}{r}\;r^2\,\frac{R}{r}\qquad \frac{2mE}{\hbar^2}\quad \frac{2mV(r)}{\hbar^2}\quad \frac{l(l\quad 1)}{r^2}\;R\quad 0$$

of the hydrogen atom, where the symbols have their usual meanings. Show that the energy values are exactly the same as those obtained by Bohr. 9+1=10

4. *(a)* Describe Gram-Schmidt ortho-normalization process. Apply this process for a doubly degenerate system. 4+2=6

(b) Consider three elements from the vector space of real 2 2 matrices :

$$|1\quad \begin{matrix}0 & 1\\ 0 & 0\end{matrix}\;,\quad |2\quad \begin{matrix}1 & 1\\ 0 & 1\end{matrix}$$

$$|3\quad \begin{matrix}2 & 1\\ 0 & 2\end{matrix}$$

Show whether they are linearly dependent or not. 4

Or

(a) Define the basis and dimensions of a vector space. 2

(b) Consider the following two kets :

$$|\qquad \begin{matrix}5i\\ 2\\ i\end{matrix}\quad \text{and}\ |\qquad \begin{matrix}3\\ 8i\\ 9i\end{matrix}$$

(i) Find | and |.

(ii) Is | normalized? If not, normalize it.

(iii) Are | and | orthogonal? 1+2+1=4

(c) Consider the state

$$|\qquad 3i\,|\,v_1\quad 7i\,|\,v_2$$

$$|\qquad |\,v_1\quad 2i\,|\,v_2$$

where $|v_1$ and $|v_2$ are orthonormal.

(i) Calculate | and |.

(ii) Show that | | . 2+2=4

5. *(a)* Calculate Bohr magneton for an electron moving in an elliptical orbit of an area *A* and time period *T*. 4

(b) Describe Stern-Gerlach experiment. 6

Or

(a) Using the eigenstates

$$\left|\frac{1}{2}, \frac{1}{2}\right\rangle \text{ and } \left\langle\frac{1}{2}, \frac{1}{2}\right|$$

as basis vectors, obtain the Pauli spin matrices. Hence, prove the commutation relation

$$[\ _i,\ \ _j]\ \ 2i\ _{ijk}\ \ _k$$

where

$$_{ijk}\ \begin{matrix} 1\ , & ijk \text{ have cyclic permutation} \\ 1\ , & ijk \text{ have anticyclic permutation} \\ 0\ , & \text{otherwise} \end{matrix}$$

6+2=8

(b) A particle with spin $\frac{1}{2}$ is in the state $\frac{1}{\sqrt{6}}\begin{matrix} 1 & i \\ 2 & \end{matrix}$, if we measure S_z. What are the probabilities of getting $\frac{h}{2}$ and $\frac{h}{2}$? 2

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Subject Code : V/PHY (vi) (R)

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce / ....) Exam., **2016**

Subject ....
Paper ....

To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce / ....) Exam., **2016**

Roll No. ....
Regn. No. ....

Subject ....
Paper ....

Descriptive Type
Booklet No. B ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

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(5th Semester)

PHYSICS

SIXTH PAPER

(Quantum Mechanics—II)

(Revised)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 10)

Put a Tick (✓) mark against the correct answer in the brackets provided : 1×10=10

1. The calculated value of the wavelength of electron using de Broglie's formula is

(*a*) 0·661 nm ( )

(*b*) 0·166 nm ( )

(*c*) 0·616 nm ( )

(*d*) 1·66 nm ( )

2. For non-dispersive medium, group velocity and phase velocity are related by the equation

(a) $v_g < v_p$ ( )

(b) $v_g = v_p$ ( )

(c) $v_g > v_p$ ( )

(d) $\frac{v_g}{v_p} = 0$ ( )

3. The eigenvalue of the particle inside a box is given by the relation

(a) $E_n = \frac{nh}{8mL^2}$ ( )

(b) $E_n = \frac{n^2h}{8m^2L}$ ( )

(c) $E_n = \frac{n^2h^2}{8mL^2}$ ( )

(d) $E_n = \frac{n^2h^2}{8m^2L^2}$ ( )

4. The potential function of a potential step is defined by

(a) $V(x) = 0 \quad , \quad x < 0$

$= V_0 \quad , \quad x > 0$ ( )

(b) $V(x) = 0 \quad , \quad x > 0$

$= V_0 \quad , \quad x < 0$ ( )

(c) $V(x) = V_0 \quad , \quad x < 0$

$= 0 \quad , \quad x = \infty$ ( )

(d) $V(x) = 1 \quad , \quad x < 0$

$= \infty \quad , \quad x > 0$ ( )

5. The energy of a harmonic oscillator is quantized in steps of

(a) $h^2\nu^2$ ( )

(b) h / ν ( )

(c) $\frac{ch}{\nu}$ ( )

(d) $h\nu$ ( )

6. The solution of the azimuthal wave equation

$$\frac{d^2\psi_3}{d\phi^2} + m_l^2\,\psi_3 = 0$$

is

(a) $\psi_3(\phi) = A\exp(im_l\,\phi)$ ( )

(b) $\psi_3(\phi) = A^2\exp(im_l\,\phi)$ ( )

(c) $\psi_3(\phi) = A\exp(im_l\,\phi + 2\pi)$ ( )

(d) $\psi_3(\phi) = A\exp\left(im_l\,\phi + \frac{2}{3}\pi\right)$ ( )

7. The angular momentum operator is defined as

(a) $L = -\frac{\vec{r}\times\vec{\nabla}}{i\hbar}$ ( )

(b) $L = -i\hbar\vec{r}\times\vec{\nabla}^2$ ( )

(c) $L = -i\hbar\vec{r}\times\vec{\nabla}$ ( )

(d) $L = -\frac{i\hbar}{\vec{r}\times\vec{\nabla}}$ ( )

8. The smallest unit of magnetic dipole moment is

(a) Bohr radius ( )

(b) Landé’s splitting factor ( )

(c) farad ( )

(d) Bohr electron magneton ( )

9. The eigenvalues of Hermitian operator are

(a) not real ( )

(b) real ( )

(c) zero ( )

(d) infinite ( )

10. The scalar product of two vectors $|\psi_1\rangle$ and $|\psi_2\rangle$ is defined by

(a) $\langle\psi_1|\psi_2\rangle = \int_{-\infty}^{\infty} \psi_1^*(x)\,\psi_2^*(x)\,dx$ ( )

(b) $\langle\psi_1|\psi_2\rangle = \int_{-\infty}^{\infty} \psi_2^*(x)\,\psi_1^*(x)\,dx$ ( )

(c) $\langle\psi_1|\psi_2\rangle = \int_{-\infty}^{\infty} \frac{\psi_2^*(x)}{\psi_1^*(x)}\,dx$ ( )

(d) $\langle\psi_1|\psi_2\rangle = \int_{-\infty}^{\infty} \frac{\psi_1^*(x)}{\psi_2^*(x)}\,dx$ ( )

SECTION—II

(*Marks* : 15)

Give short answers to the following questions : 3×5=15

1. What is expectation value of an operator? Obtain the expectation value for energy and momentum.

2. Give the physical interpretation of wave function. What does normalization condition mean?

3. What are the three quantum numbers associated with wave functions of a hydrogen atom? Give their significance.

4. What are linear vector space and Hilbert space?

5. Show that $[L_z, L^2]$ is equal to zero, where L is angular momentum operator.

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