2017

(5th Semester)

PHYSICS

FIFTH PAPER

(Mathematical Physics—I)

(Pre-Revised)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. *(a)* What do you mean by singular point of a differential equation? Find the singular points for the following differential equations : 3

(i) $(x \quad 5)y \quad (x \quad 1)y \quad 0$

(ii) $(x^2 \quad 1)y \quad y \quad x^2 y \quad 0$

(b) Using Frobenius method, find the series solution about $x \quad 0$ of $x(x \quad 1)y \quad 3xy \quad y \quad 0$. 7

Or

(c) Show that the solution of the partial differential equation of heat flow in one-dimension with initial conditions $u(x, 0) \quad \cos 2x$ is $u(x, t) \quad \exp(\ 4t)\cos 2x$, where $u(x, t)$ is the temperature distribution function. 8

(d) Write down two-dimensional Laplace's equation in both Cartesian and Polar coordinates defining the symbols (coordinate variables) used in the equations. 2

2. *(a)* Prove the following recursion relations for Legendre polynomials : 7

(i) $nP_n(x) \quad (2n \quad 1)xP_{n \quad 1}(x) \quad (n \quad 1)P_{n \quad 2}(x)$

(ii) $nP_n(x) \quad xP_n(x) \quad P_{n \quad 1}(x)$

(b) For Hermite's polynomial $H_n(x)$, show that $2xH_n(x) \quad 2nH_{n \quad 1}(x) \quad H_{n \quad 1}(x)$ for $n \quad 1$. 3

Or

(c) Prove the following : 4

(i) $H_{2n}(0) \quad (\ 1)^n \frac{(2n)!}{n!}$

(ii) $H_{2n \quad 1}(0) \quad 0$

(d) Show that

$$J_{3/2}(x) \quad \sqrt{\frac{2}{x}} \frac{\sin x}{x} \quad \cos x$$ 6

3. *(a)* Write down Cauchy-Riemann conditions for the analyticity of a function of complex variables. Find out whether or not the function $f(z)$ u iv with $u(x, y)$ x^2 y^2 and $v(x, y)$ $2xy$ satisfy Cauchy-Riemann equations. 4

(b) Use residue theorem to evaluate the integral

$$\frac{dx}{x^2 \quad 2x \quad 2}$$ 6

Or

(c) Find the Laurent series expansion of $f(z)$ $\frac{2z^3 \quad 1}{z^2 \quad z}$ about z 1. 5

(d) What do you mean by singularity? Find the location of singularities of the following : 5

(i) $\frac{1}{\cos z \quad \sin z}$

(ii) $\frac{1}{1 \quad e^z}$

4. *(a)* What do you understand by orthogonal curvilinear coordinate system? Deduce the expression for the gradient of a scalar field in general orthogonal curvilinear coordinate system. 1+5=6

(b) A central force field is given by

$$\vec{F} \quad \hat{e}_r \frac{2r_o \cos}{r^3} \quad \hat{e} \quad \frac{r_o}{r^3} \sin$$

Calculate $\vec{} \quad \vec{F}$. 4

Or

(c) What are covariant, contravariant and mixed tensors? Show that Kronecker delta is a mixed tensor of rank 2. 2+2=4

(d) If $a \quad x \quad x \quad 0$ for all values of the variables $x^1, x^2, \cdots, x^N$. Show that $a_{ij} \quad a_{ji} \quad 0$. 3

(e) If A and B are tensors, then prove that their sum and difference are also tensors of the same type. 3

5. *(a)* Show that every square matrix can be uniquely expressed as a sum of a symmetric and a skew-symmetric matrices. 3

(b) What is unitary matrix? Show that the matrix

$$\begin{matrix} 0 & i \\ i & 0 \end{matrix}$$

is unitary. 3

(c) Show that any two eigenvectors corresponding to two distinct eigenvalues of a Hermitian matrix are orthogonal. 4

Or

(d) Find the eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} \cos & \sin \\ \sin & \cos \end{pmatrix}$$ 5

(e) Diagonalize the matrix

$$\begin{pmatrix} \cos & \sin & 0 \\ \sin & \cos & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ 5

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Subject Code : PHY/V/05 (PR)

Booklet No. A

Date Stamp .....

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To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce / .....) Exam., **2017**

Subject .....
Paper .....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce / .....) Exam., **2017**

Roll No. .....
Regn. No. .....

Subject .....
Paper .....

Descriptive Type
Booklet No. B .....

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

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(5th Semester)

PHYSICS

FIFTH PAPER

(Mathematical Physics—I)

(Pre-Revised)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 10)

Put a Tick ☑ mark against the correct answer in the box provided : 1×10=10

1. Which of the following is a wave equation?

(a) $\frac{^2u}{x^2}$ $c^2\frac{^2u}{t^2}$ ☐

(b) $\frac{^2u}{x^2}$ $\frac{1}{c^2}\frac{^2u}{t^2}$ ☐

(c) $\frac{u}{t}$ $c^2\frac{^2u}{x^2}$ ☐

(d) ^2u k^2u 0 ☐

2. The solution of the differential equation $y \quad e^{2x}$ is

(a) $y \quad e^{x} \quad c$ ☐

(b) $y \quad e^{2x} \quad c$ ☐

(c) $y \quad \frac{e^{2x}}{2} \quad c$ ☐

(d) $y \quad e^{2x} \quad 4$ ☐

3. The value of $\int_{1}^{1}(2x \quad 1)P_3(x)\,dx$ is

(a) 1 ☐

(b) –1 ☐

(c) 2 ☐

(d) 0 ☐

4. The value of $\frac{J_{1/2}(x)}{J_{\ 1/2}(x)}$ is

(a) 1 ☐

(b) $\tan x$ ☐

(c) $\cot x$ ☐

(d) $\tanh x$ ☐

5. The order of the pole of the function $f(z)$ $\dfrac{z^4 \quad 2z \quad 1}{z^2 \quad 5z \quad 1}$ at z is

(a) 2 ☐

(b) 1 ☐

(c) 0 ☐

(d) 4 ☐

6. The value of the integral $_C \tan z \, dz$, where C is $|z|$ 2 is

(a) 0 ☐

(b) i ☐

(c) 2 i ☐

(d) $\frac{}{2}$ ☐

7. In three-dimensional Cartesian system, the value of dirac delta function $_{ii}$ is

(a) 0 ☐

(b) 1 ☐

(c) 2 ☐

(d) 3 ☐

8. In spherical coordinate system $(r, \ , \ )$, the intersection between coordinate surfaces $r \quad c_1$ and c_2 is

(a) a straight line ☐

(b) a circle ☐

(c) a semicircle ☐

(d) an ellipse ☐

9. If $A^T A \quad I$, the square matrix A is called

(a) Hermitian matrix ☐

(b) unitary matrix ☐

(c) orthogonal matrix ☐

(d) symmetric matrix ☐

10. For the Hermitian matrices A and B,

(a) $(AB)^\dagger \quad B^\dagger A^\dagger$ ☐

(b) $(AB)^\dagger \quad A^\dagger B^\dagger$ ☐

(c) $(AB)^T \quad B^T A^T$ ☐

(d) $A^\dagger A \quad B^\dagger B$ ☐

SECTION—II

(*Marks* : 15)

Write short answers to the following questions : 3×5=15

1. Explain the difference between an ordinary differential equation and a partial differential equation. Give examples.

2. Using the recursion relation,

$$\frac{d}{dx}[x^n J_n(x)] \quad x^n J_{n\ 1}(x)$$

show that

$$\int_0^x x^n J_{n\ 1}(x)\, dx \quad x^n J_n(x)$$

3. Test the analyticity of $|z|$, where $z \quad x \quad iy$.

4. Write the coordinate transformation relations for the following tensors :

(i) A_l^k

(ii) A_k^{ij}

5. If H is a Hermitian matrix, show that e^{iH} is unitary matrix.

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