

2 0 1 6

(5th Semester)

PHYSICS

FIFTH PAPER

(Mathematical Physics—I)

(Pre-revised)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(Marks : 50)

*The figures in the margin indicate full marks
for the questions*

1. (a) Explain the meaning of ordinary point of differential equation by giving an example. 2
- (b) Find the power series solution of $(1 - x^2)y - 2xy - 2y = 0$ about $x = 0$. 8

Or

Use the method of separation of variables to solve the equation $\frac{v}{x^2} = \frac{v}{t}$. Given that $v = 0$ at $x = 0$ and $x = l$, $v = 0$ at $t = \dots$. 10

2. (a) Show that $P_n(x) = \frac{1}{2^n (n!)} \frac{d^n}{dx^n} (x^2 - 1)^n$. 5

(b) Express the function

$$f(x) = 4x^3 - 6x^2 - 7x + 2$$

in terms of Legendre's polynomials. 5

Or

- (a) For Hermite polynomials $H_n(x)$, show that $H_n(x) = 2nH_{n-1}(x)$. 3

(b) Prove the following : $3\frac{1}{2} + 3\frac{1}{2} = 7$

$$(i) xJ_n(x) - nJ_n(x) = xJ_{n-1}(x)$$

$$(ii) 2J_n(x) - J_{n-1}(x) - J_{n+1}(x)$$

3. (a) What do you mean by Argand diagram? What is the modulus of the complex number $a + ib$? 2
- (b) Deduce the Cauchy-Reimann conditions for the analyticity of a function of complex variable. 4
- (c) State and prove Cauchy integral theorem. 4

(3)

Or

- (a) Find the first four terms of the Taylor's series expansion of the complex variable function

$$f(z) = \frac{z-1}{(z-3)(z-4)}$$

about $z=2$. 5

- (b) By using residue theorem, show that

$$\oint_0^{2\pi} \frac{dz}{2 - \cos \frac{z}{\sqrt{3}}} = 5$$

4. (a) Discuss the two sets of unit vectors in a curvilinear coordinate system (u_1, u_2, u_3) . 4

- (b) Deduce the expressions for the divergence and curl of a vector field in cylindrical coordinates. 6

Or

- (a) What are symmetric and skew-symmetric tensors? Show that every tensor of rank 2 can be expressed as the sum of symmetric and skew-symmetric tensors. 2+3=5

- (b) If A and B are the components of a contravariant and covariant tensors of rank 1 each, show that $C = A B$ are the components of a mixed tensor of rank 2. 3

(4)

- (c) Write the transformation relation for the following tensors : 2

(i) A and (ii) A

5. (a) Show that the transpose of the product of two matrices is the product of their transposes taken in the reverse order, i.e., $(AB)^T = B^T A^T$, where T denotes transpose. 3

- (b) Solve the following simultaneous equations by matrix method : 5

$$\begin{matrix} 2x & 3y & 4z & 9 \\ & 2y & 3z & 8 \\ & & x & z & 3 \end{matrix}$$

- (c) What are orthogonal and unitary matrices? 2

Or

- (a) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$. 5

- (b) The matrix $A = \begin{pmatrix} a & h \\ h & b \end{pmatrix}$ is transformed to the diagonal form $D = T^{-1} A T$, where

$$T = \begin{pmatrix} \cos & \sin \\ \sin & \cos \end{pmatrix}$$

Find the value of θ , which gives the diagonal transformation. 5

Subject Code : **V**/PHY (v) (PR)

Booklet No. **A**

Date Stamp

.....

To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce /
.....) Exam., **2016**

Subject

Paper

**To be filled in by the
Candidate**

DEGREE 5th Semester
(Arts / Science / Commerce /
.....) Exam., **2016**

Roll No.

Regn. No.

Subject

Paper

Descriptive Type

Booklet No. B

INSTRUCTIONS TO CANDIDATES

1. The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.
2. This paper should be **ANSWERED FIRST** and submitted within **1 (one) Hour** of the commencement of the Examination.
3. While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.

Signature of
Scrutiniser(s)

Signature of
Examiner(s)

Signature of
Invigilator(s)

/127

V/PHY (v) (PR)

2 0 1 6

(5th Semester)

PHYSICS

FIFTH PAPER

(Mathematical Physics—I)

(Pre-revised)

(PART : A—OBJECTIVE)

(Marks : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(Marks : 10)

Put a Tick (✓) mark against the correct answer in the
brackets provided : 1×10=10

1. If a function of two variables is a solution of Laplace's equation, the function is said to be

- (a) conjugate ()
- (b) harmonic ()
- (c) anharmonic ()
- (d) discontinuous ()

/127

(2)

2. In the differential equation

$$(1 - x^2)y'' - 2xy' + l(l+1)y = 0$$

- (a) $x = 0$ is an ordinary point ()
- (b) $x = \pm 1$ are regular singular points ()
- (c) Both (a) and (b) are correct ()
- (d) $x = \pm 2$ are regular singular points ()

3. For integral values of n , $J_n(-x)$ is

- (a) $(-1)^n J_n(x)$ ()
- (b) $J_n(x)$ ()
- (c) 0 ()
- (d) $(-1)^n \pi J_n(x)$ ()

4. The value of $H_2(x)$ is

- (a) $x^2 - 1$ ()
- (b) $2x^2 - 1$ ()
- (c) $4x^2 - 2$ ()
- (d) $\frac{1}{2}(3x^2 - 1)$ ()

(3)

- 5.** The equation for a circle with centre at $(-1, 1)$ and radius 3 is

(a) $|z + 1 + i| = 3$ ()

(b) $|z + 1 - i| = 3$ ()

(c) $|z - 1 - i| = 3$ ()

(d) $|z - 1 + i| = 3$ ()

- 6.** The function $\frac{1}{(z-1)^{1/2}}$

(a) is analytic in the region $|z| < 2$ ()

(b) has a pole at $z = 1$ ()

(c) has a branch point at $z = 1$ ()

(d) has an essential singularity at $z = 1$ ()

(4)

7. Rotation of a point (0, 1) using the matrix

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

about the origin through an angle $\frac{\pi}{4}$ results in a point

(a) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ ()

(b) $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ ()

(c) (1, 0) ()

(d) (0, -1) ()

8. Rank of $n \times m$ matrix is the

(a) largest square submatrix with non-zero determinant ()

(b) number of columns in $n \times m$ matrix ()

(c) number of rows in $n \times m$ matrix ()

(d) largest square submatrix with zero determinant ()

V/PHY (v) (PR)/127

(5)

9. In cylindrical coordinate system, the intersection between coordinate surfaces $\rho = c_1$ and $z = c_3$ is a/an

(a) straight line ()

(b) circle ()

(c) semicircle ()

(d) ellipse ()

10. If x^i , $i = 1, 2$ represents rectangular coordinates, the relation $x^i x^i = 1$ would represent

(a) a circle of unit radius ()

(b) a sphere of unit radius ()

(c) a straight line in two dimensions ()

(d) a hypersphere of unit radius ()

V/PHY (v) (PR)/127

(6)

SECTION—II

(Marks : 15)

Give short answers to the following questions : 3×5=15

1. Find the differential equation for which the solution is $y = c_1e^x + c_2e^{-x} + 3x$.

(7)

2. Using Rodrigues formula, prove that

$$\int_{-1}^{+1} P_0(x) dx = 2$$

(8)

3. Using residue theorem, evaluate the integral $\int_C \frac{1+z}{z(2-z)} dz$, where the circle C is $|z|=1$.

(9)

4. Explain Einstein's summation convention. Give an example.

(10)

5. What do you mean by row, column and symmetric matrices? Give one example each.
