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(6th Semester)

MATHEMATICS

Paper : Math–363

(Mechanics)

Full Marks : 75

Time : 3 hours

(Part : B—descriptive)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

Unit—I

1. *(a)* Forces equal to $3P$, $7P$ and $5P$ act along the side AB, BC and CA of an equilateral triangle ABC. Find the magnitude, direction and line of action of the resultant. 5

(b) Determine how high can a particle rest inside a rough hollow sphere of radius a, if the coefficient of friction is $\frac{1}{\sqrt{3}}$. 5

2. *(a)* A uniform rod rests within a fixed vertical subtending an angle 2 at the centre, its upper end is smooth and its lower end is rough. Show that the angle which the rod makes with horizon cannot be greater than , where

$$\tan \qquad \frac{\sin}{\cos \qquad \cos(\quad 2 \quad)}$$ 5

(b) A hemisphere of radius a and weight W is placed with its curved surface on a smooth table and a string of length $l(\quad a)$ is attached to a point on its rim and a point on the table. Find the position of equilibrium and prove that the tension of the string is

$$\frac{3W}{8} \frac{a \quad l}{\sqrt{2al \quad l^2}}$$ 5

Unit—II

3. *(a)* State and prove perpendicular axes theorem on moments of inertia. 5

(b) A thin uniform wire is bent into the form of a triangle ABC and heavy particles of weights P, Q, R are placed at the angular points. If the centre of mass of the particles coincides with that of the wire, then prove that

$$\frac{P}{b \quad c} \quad \frac{Q}{c \quad a} \quad \frac{R}{a \quad b}$$ 5

4. *(a)* If a piece of wire is bent into a shape of an isosceles triangle, whose equal sides are a and b, show that the distance of the CG from the base is

$$\frac{a}{2}\sqrt{\frac{2a \quad b}{2a \quad b}}$$ 5

(b) From a circular disc of radius r, a circle is cut out such that its diameter is the radius of the disc. Find the CG of the remaining disc. 5

UNIT—III

5. *(a)* Prove that if the tangential and normal component of acceleration of a particle describing a plane curve be constant throughout the motion, the angle through which the direction of motion turns in time t is given by

$$A \log(1 \quad Bt)$$

where A and B are constants. 5

(b) A curve is described by a particle having constant acceleration in a direction inclined at a constant angle to the tangent. Show that the curve is an equiangular spiral. 5

6. *(a)* A train travels a distance s in t second. It starts from rest and ends at rest. In the first part of the journey it moves with a constant acceleration f and in the second part with constant retardation f . Show that if s is the distance between the two stations, then

$$t \quad \sqrt{2s \quad \frac{1}{f} \quad \frac{1}{f}}$$ 5

(b) A particle rests in equilibrium under the attraction of two centres of forces which attract directly as the distance between the two centres, their intensities being , . The particle is displaced slightly towards one of them. Show that the time of small oscillation is

$$\frac{2}{\quad}$$ 5

UNIT—IV

7. *(a)* If t be the time in which a projectile reaches a point P in its path and t the time from P till it reaches the horizontal plane through the point of projection, show that the height of P above the horizontal plane is $\frac{1}{2}gtt$. Also prove that the greatest height of the projectile is $\frac{1}{8}g(t \quad t)^2$. 5

(b) A body is projected at an angle   to the horizon, so as just to clear two walls of equal height a at a distance $2a$ from each other. Show that the range is equal to

$$2a\cot\frac{\ }{2}$$ 5

8. *(a)* From a point in a given inclined plane, two bodies are projected with the same velocity in the same vertical plane at right angles to one another. Show that the difference of their ranges is constant. 5

(b) A particle of mass m is falling under the influence of gravity through a medium whose resistance equals   times the velocity. If the particle is released from rest, then show that the distance fallen through in time t is

$$g\frac{m^2}{\ ^2}\quad e^{\frac{t}{m}}\quad 1\quad \frac{t}{m}$$ 5

UNIT—V

9. *(a)* A shot of mass is projected from a gun of mass M by an explosion which generates a kinetic energy E. Show that the gun recoil with a velocity

$$\sqrt{\frac{2mE}{M(M\quad m)}}$$

and the initial velocity of the shot is

$$\sqrt{\frac{2mE}{m(M\quad m)}}$$ 5

(b) The earth's attraction on a particle varies inversely as the square of its distance from the earth's centre. A particle whose weight on the surface of the earth is W, falls to the surface of the earth from a height $5a$ above it. Show that the work done by the earth's attraction is

$$\frac{5aW}{6}$$

where a is the radius of the earth. 5

10. *(a)* A smooth sphere of mass m, travelling with a velocity u, impinges obliquely on a smooth sphere of mass M at rest, the original line of motion of the first sphere making an angle   with the line of centres at the moment of the impact. If the coefficient of restitution be e, show that the impinging sphere will be deflected through a right angle, if

$$\tan^2 \qquad \frac{eM\quad m}{M\quad m}$$

and that its velocity perpendicular to its original line of motion will be obtained, if   45°. 5

(b) A sphere impinges directly on an equal sphere which is at rest. Show that a fraction

$$\frac{1}{2}(1 \quad e^2)$$

of the original kinetic energy is lost during the impact. 5

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Subject Code : MATH/VI/11

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 6th Semester
(Arts / Science / Commerce / ....) Exam., **2017**

Subject ....

Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

To be filled in by the Candidate

DEGREE 6th Semester
(Arts / Science / Commerce / ....) Exam., **2017**

Roll No. ....

Regn. No. ....

Subject ....

Paper ....

Descriptive Type
Booklet No. B ....

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

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2 0 1 7

(6th Semester)

MATHEMATICS

Paper : Math–363

(Mechanics)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Put a Tick ☑ mark against the correct answer in the box provided : 1×10=10

1. The least force *P* required to pull a body down an inclined plane inclined at an angle to the horizontal is attained, when (where is the angle of friction and is the angle made by the force *P* to the inclined plane)

(a) ☐

(b) ☐

(c) ☐

(d) ☐

2. Suppose that a system of forces acts at different points of a rigid body is in equilibrium, then

(a) the resultant R must vanish ☐

(b) the moment of all the forces w.r.t. each of three collinear points is zero ☐

(c) the couple G must vanish ☐

(d) the resultant R and the couple G must separately vanish ☐

3. The centre of gravity of three uniform rods forming a triangle is at

(a) the incenter of the triangle ☐

(b) the centroid of the triangle ☐

(c) the orthocenter of the triangle ☐

(d) None of the above ☐

4. The moment of inertia of a plane distribution with respect to any normal axis

(a) is equal to its moment of inertia ☐

(b) cannot be determined ☐

(c) is equal to the sum of the moments of inertia ☐

(d) is equal to the product of inertia ☐

5. If the position of a moving particle at time t referred to rectangular axes is given by $x \quad at$, $y \quad bt \quad ct^2$, where a, b and c are constants, then its acceleration at time t is

(a) along the x-axis ☐

(b) $a \quad b \quad c$ along the x-axis ☐

(c) $\sqrt{a^2 \quad b^2}$ along the y-axis ☐

(d) $2c$ along the y-axis ☐

6. If a particle moves so that its normal acceleration is always zero, then its path is

(a) a circle ☐

(b) a parabola ☐

(c) a straight line ☐

(d) None of the above ☐

7. If a particle moves along the x-axis under an attraction towards the origin O, varying inversely as the square of the distance from it, then the equation of motion is

(a) $x \quad \frac{}{x^2}$ ☐

(b) $x \quad \frac{}{x^2}$ ☐

(c) $x \quad x^2$ ☐

(d) $x \quad x^2$ ☐

8. If a particle is projected with a velocity u from the ground at an angle with the horizontal, then the velocity of the particle at height h is

(a) $\sqrt{u^2 \quad 2gh}$ ☐

(b) $u^2 \quad 2gh$ ☐

(c) $\sqrt{u \quad h\sin}$ ☐

(d) None of the above ☐

9. A smooth sphere of mass m strikes a plane normally and is rebounded. If e is the coefficient of restitution, then the loss of kinetic energy is

(a) $\frac{1}{2}me^2u^2$ ☐

(b) $\frac{1}{2}m(1 \quad e^2)u^2$ ☐

(c) $\frac{1}{2}m(1 \quad e^2)u^2$ ☐

(d) None of the above ☐

10. A sphere falling vertically from a height h impinges on a horizontal fixed table and rebounds to a height h_1. If e is the coefficient of restitution between the sphere and the plane, then

(a) $h_1 \quad e^2h$ ☐

(b) $h_1 \quad 2e^2h$ ☐

(c) $h_1 \quad \sqrt{eh}$ ☐

(d) None of the above ☐

SECTION—B

(*Marks* : 15)

Answer the following questions : $3 \times 5 = 15$

1. Prove that if three forces, acting in one plane upon a rigid body, keep it in equilibrium, then they must either meet in a point or be parallel.

2. Prove that the centre of gravity of a triangular area coincides with that of three equal particles placed at the middle points of its sides.

3. A particle moves in a plane with constant speed. Prove that its acceleration is perpendicular to its velocity.

4. Find the greatest height attained by the projectile thrown with a velocity u at an angle with the horizontal.

5. A billiard ball impinges directly on another equal ball at rest. If e is the coefficient of restitution, prove that their velocities after impact are in the ratio $(1 \quad e):(1 \quad e)$.

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