Subject: Mathematics

Paper name: Vector Calculus and Solid Geometry

Paper No:MATH/IV/EC/241

Semester: Fourth Semester

1. The value of $[\ \hat{i}\ \hat{k}\ \hat{j}\ ]$ is

(a) 1

(b) -1

(c) 0

(d) 2

2. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be any three vectors , then $\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})$ is

(a) $\vec{0}$

(b) $\vec{a}$

(c) $\vec{b}$

(d) $\vec{c}$

3. The projection of $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$ is

(a) $\frac{5}{6}(2\hat{i}-\hat{j}+\hat{k})$

(b) $\frac{6}{5}(2\hat{i}-\hat{j}+\hat{k})$

(c) $\frac{6}{5}(\hat{i}-2\hat{j}+\hat{k})$

(d) $\frac{5}{6}(\hat{i}-2\hat{j}+\hat{k})$

4. If $\left|\vec{a}\right|$=4 , $\left|\vec{b}\right|$=5 and $\vec{a}.\vec{b}=0$, then $\vec{a}\times\vec{b}$ is

(a) $20\hat{n}$

(b) $9\hat{n}$

(c) $\hat{n}$

(d) 0

5. The set of three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are said to be coplanar when

(a) [$\vec{a}\ \vec{b}\ \vec{c}$] = 0

(b) [$\vec{a}\ \vec{b}\ \vec{c}$] $\neq$ 0

(c) [$\vec{a}\ \vec{b}\ \vec{c}$] > 0

(d) None of the above

6. If $\vec{f}=(2x+y)\hat{i}-(3y+2z)\hat{j}+(x+az)\hat{k}$ is solenoidal , then the value of a is

(a) 0

(b) 1

(c) 2

(d) -2

7. The directional derivative of $\phi = xy + yz + zx$ at (1,2,0) in the direction of $\hat{i} + 2\hat{j} + 2\hat{k}$ is

(a) $\frac{3}{4}$

(b) $\frac{4}{3}$

(c) $\frac{10}{3}$

(d) 10

8. A particle moves along a curve whose parametric equations are

$x = e^{-t}, y = 2\cos 3t, z = \sin 3t$, then the acceleration of particle at $t = 0$ is

(a) $\sqrt{325} unit/\sec^2$

(b) $\sqrt{10} unit/\sec^2$

(c) $\sqrt{275} unit/\sec^2$

(d) $\sqrt{429} unit/\sec^2$

9. If $\vec{A} = 2yz\hat{i} - x^2y\hat{j} + xz^2\hat{k}$ and $\phi = 2x^2yz^3$ then $(\vec{A}.\nabla)\phi$ equals to

(a) $8xy^2z^4 - 2x^4yz^3 + 6x^3yz^4$

(b) $8xy + xz - y^3$

(c) $8xy^3 + xy + 6x^3y$

(d) None of the above

10. By Stoke's theorem the value of $\iint_S \left(\nabla \times \vec{A}\right).\vec{n}\,dS$ where

$\vec{A} = \left(x^2 + y - 4\right)\hat{i} + 3xy\,\hat{j} + \left(2xz + z^2\right)\hat{k}$ and S is the surface of the paraboloid $z = 4 - \left(x^2 + y^2\right)$ above the xy-plane is

(a) 3π

(b) -4π

(c) 4π

(d) -3π

11. Let y=mx+c and y'=m'x+c' are the two equation of straight line, then the angle between the two lines is

(a)$\tan^{-1}\left(\frac{m+m'}{1+mm'}\right)$

(b) $\tan^{-1}\left(\frac{m-m'}{1+mm'}\right)$

(c) $\tan^{-1}\left(\frac{m+m'}{1-mm'}\right)$

(d) $\tan^{-1}\left(\frac{m-m'}{1-mm'}\right)$

12. Let m and m' be the slope of two lines then the two lines are parqallel if

(a)mm'=1

(b) m=-m'

(c) m=m'

(d)m=$-\frac{1}{m'}$

13. The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$represent an ellipse if

(a) $ab-h^2>0$
(b) $ab-h^2=0$
(c) $ab-h^2<0$
(d) a=b and h=0

14. the asymptotes of the hyperbola xy+4x+3y+15 is

(a) (x+4)(y+3)=0
(b) (x+3)(y+4)=0

(c) (x+3)(y-4)=0
(d) (x+4)(y-3)=0

15. The common asymptotes of the hyperbola $4x^2 + 3xy + 5x + 21 = 0 \; and \; x^2 - 4xy - 3x + 19 = 0$ is

(a) 4x+3y+5=0
(b) x-4y-3=0
(c) x=0
(d) None of the above

16. The equation of the plane passing through (2,3,5) parallel to the plane 2x-4y+3z=9 is

(a) 2x+4y+3z-5=0
(b) 2x+4y+3z-7=0
(c) 2x-4y+3z+5=0
(d) 2x-4y+3z-7=0

17. Shortest distance between the line $\frac{x-3}{1} = \frac{y-4}{1} = \frac{z+1}{-3}$ and $\frac{x-1}{-1} = \frac{y-3}{3} = \frac{z-1}{2}$ is

(a) $15/\sqrt{79}$
(b) $15/\sqrt{138}$
(c) $13/\sqrt{138}$
(d) $13/\sqrt{79}$

18. The intercept on the x axis by the plane 3x-6y+12z=12 is

(a) 1
(b) 2
(c) 3
(d) 4

19. Shortest distance between the line $\frac{x-1}{5} = \frac{y-7}{-4} = \frac{z+3}{12}$ and the y-axis is

(a) 27/13
(b) 13/27
(c) 13/25
(d) 27/15

20. The angle between the plane x+y+z=1 and x-y=2 is

(a) 0
(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{3}$

21. If sphere of radius r_1 and r_2 cut orthogonally then the radius of the common circle is

(a) $\frac{r_1 r_2}{\sqrt{{r_1}^2+{r_2}^2}}$

(b) $\frac{r_1 r_2}{\sqrt{{r_1}^2-{r_2}^2}}$

(c) $\frac{\sqrt{{r_1}^2-{r_2}^2}}{r_1 r_2}$

(d) $\frac{\sqrt{{r_1}^2+{r_2}^2}}{r_1 r_2}$

22. If d is the distance between the center of two sphere of radii $r_1 \, and \, r_2$, then the angle between them is

(a) $tan^{-1}\left(\frac{{r_1}^2+{r_2}^2-d^2}{2r_1 r_2}\right)$

(b) $sin^{-1}\left(\frac{{r_1}^2+{r_2}^2-d^2}{2r_1 r_2}\right)$

(c) $cos^{-1}\left(\frac{{r_1}^2+{r_2}^2-d^2}{2r_1 r_2}\right)$

(d) $none \, of \, the \, above.$

23. The equation of the sphere passing through the circle S=0,P=0 is

(a) S+λp>0

(b) S+λp=0

(c) S+λp<0

(d) None of the above

24. Equation of the right circular cylinder wit radius *a* and y axis as the axis of the cylinder $x^2 + z^2 = a^2$

(a) $\sqrt{x^2 + z^2} = a$

(b) $\sqrt{x^2 + z^2} = a^2$

(c) $x^2 + z^2 = a^2$

(d) None of the above

25. The sum of the square of the intercept made by a given sphere on any three mutually perpendicular straight line through a fixed point is

(a) Maximum

(b) Minimum

(c) Varies

(d) Constant

B. Fill

1. For any two proper vectors $\vec{a}$ and $\vec{b}$ i.e. $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}, \vec{a}.\vec{b} = 0 \Leftrightarrow$ _____ .

2. A necessary and sufficient condition that a proper vector $\vec{u}$ has a constant length is _____ .

3. If $\vec{r(t)} = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$ is the position vector of a particle moving along a smooth curve in space at time t , then velocity at time t , $\vec{v(t)}$ = _____ .

4. A vector $\vec{A}$ is said to be irrotational if _____ .

5. If $\phi(x, y, z) = x^2y^2 + xz^3$ then $grad\phi$ = _____.

6. If $\vec{A}$ is a differentiable vector function and ϕ is a differentiable scalar function , then $\nabla.\left(\phi\vec{A}\right) =$ _____ .

7. The equation of the circle $(x-2)^2+(y-3)^2=r^2$ when the origin is transfer to(2,1) is _____.

8. The equation of the curves 2x+5y=15 when the origin is shifted to (5,6) is _____.

9. The transformed equation of $x^2-y^2=a^2$ by rotating an angle 45 to the original axes without changing the axes is _____.

10. The equation of the plane passing through (2,3,5) parallel to the plane 2x-4y+3z=9 is _____.

11. If two plane ax+by+cz+d=0and a'x+b'y+c'z+d'=0 are perpendicular if _____.

12. Shortest distance between the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-3}{5}$ is _____.

13. Equation of sphere with center (a,b,c) and radius r is _____.

14. In a right circular cone the vertex is origin ,then the equation of the cone is_____.

15. In a right circular cone the vertex is origin and x axis is the axis of the cone, then (α,β,γ)=(0,0,0) and the direction cosine of the axis be 1,0,0 ,the equation of the cone is_____.

Key Answers

Multiple choice

1. (b)	2. (a)	3. (d)	4. (a)	5. (a)	6. (b)
1. (c)	8. (a)	9. (a)	10.(b)	11.(b)	12.(c)
13. (a)	14. (b)	15. (c)	16. (d)	17.(b)	18.(d)
19. (a)	20. (b)	21. (a)	22. (c)	23. (b)	24. (c)

25. (d)

Fill in the blanks

1. $\vec{a}\perp\vec{b}$ (or $\vec{a}$ is perpendicular to $\vec{b}$)

2. $\vec{u}.\frac{d\vec{u}}{dt}=0$

3. $\vec{v(t)}=\frac{d\vec{r(t)}}{dt}$

4. $curl\,\vec{A}=0(or\nabla\times\vec{A}=0)$

5. $(2xy^2+z^3)\hat{i}+2x^2y\,\hat{j}+3xz^2\,\hat{k}$

6. $(\nabla\phi).\vec{A}+\phi\left(\nabla.\vec{A}\right)$

7. $x^2+(y-2)^2=r^2$

8. $2x+5y+25=0$

9. $2xy+a^2=0$

10. $2x-4y+3z-7=0$

11. $aa'+bb'+cc'=0$

12. 0

13. $(x-a)^2+(y-b)^2+(z-c)^2=r^2$

14. $(x^2+y^2+z^2)(l^2+m^2+n^2)\cos^2\alpha=(lx+my+nz)^2$

15. $(x^2+y^2+z^2)\cos^2\alpha=x^2$ or is $(y^2+z^2)=x^2\tan^2\alpha$