2017

(4th Semester)

MATHEMATICS

Paper : MATH–241

(Vector Calculus and Solid Geometry)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Prove that for a triangle *ABC*,

$$\frac{\sin A}{a} \quad \frac{\sin B}{b} \quad \frac{\sin C}{c}$$

where $BC \quad a$, $CA \quad b$ and $AB \quad c$. 4

(b) Find the set of vectors reciprocal to the set $\quad 2\hat{i} \quad 3\hat{j} \quad \hat{k}$, $\quad \hat{i} \quad \hat{j} \quad 2\hat{k}$ and $\hat{i} \quad 2\hat{j} \quad 2\hat{k}$. 4

(c) Find the unit tangent vector to the space curve $\vec{r} \quad t\,\hat{i} \quad t^2\hat{j} \quad \frac{2}{3}t^3\hat{k}$ at $t \quad 1$. 2

2. *(a)* Prove that the necessary and sufficient condition for the vector function $\vec{f}(t)$ to have constant direction is $\vec{f} \quad \frac{d\vec{f}}{dt} \quad \vec{0}$. 5

(b) If three concurrent edges of a parallelepiped are given by

$$\vec{a} \quad 3\hat{i} \quad 7\hat{j} \quad 5\hat{k}, \vec{b} \quad 5\hat{i} \quad 7\hat{j} \quad 3\hat{k} \text{ and } \vec{c} \quad 7\hat{i} \quad 5\hat{j} \quad 3\hat{k}$$

then find its volume. 5

UNIT—II

3. *(a)* Show that $\quad f(r) \quad \vec{r} \quad \vec{0}$, where $\vec{r} \quad x\hat{i} \quad y\hat{j} \quad z\hat{k}$ and $r \quad |\vec{r}|$. 5

(b) Let $\quad (x, y, z) \quad 3x^2y \quad y^2z^2$. Find the directional derivative of $\quad$ at $(1, \quad 2, \quad 1)$ in the direction $\hat{i} \quad \hat{j} \quad \hat{k}$ and the greatest directional derivative at that point. 4+1

4. *(a)* Find the work done in moving a particle from (0, 0) to (1, 2) along the curve $y \quad 2x^2$ in *xy*-plane, where the force field is $\vec{F} \quad 3xy\hat{i} \quad y^2\hat{j}$. 3

(b) Show that curl grad $\quad \vec{0}$, where $\quad$ is a scalar function of x, y and z. 3

(c) Verify Stokes' theorem for the function $\vec{F} \quad x^2\hat{i} \quad xy\hat{j}$ integrated along the rectangle in the xy-plane whose sides are along the lines $x \quad 0$, $y \quad 0$; $x \quad a$ and $y \quad b$. 4

UNIT—III

5. *(a)* Find the transformed equation of the curve $(x \quad 2y \quad 4)(2x \quad y \quad 5) \quad 25$, when the two perpendicular lines $x \quad 2y \quad 0$ and $2x \quad y \quad 0$ are taken as coordinate axes. 5

(b) Find the values of k for which the equation

$$3x^2 \quad kxy \quad 3y^2 \quad 29x \quad 3y \quad 18 \quad 0$$

represents a pair of straight lines. 5

6. *(a)* Reduce the equation

$$7x^2 \quad 2xy \quad 7y^2 \quad 16x \quad 16y \quad 8 \quad 0$$

to the standard form and hence show that it is the equation of an ellipse. 6

(b) Find the pair of tangents from the point (1, 1) to the conic

$$14x^2 \quad 4xy \quad 11y^2 \quad 44x \quad 58y \quad 71 \quad 0$$ 4

UNIT—IV

7. *(a)* Show that the lines

$$\frac{x \quad 2}{3} \quad \frac{y \quad 1}{1} \quad \frac{z \quad 1}{2}$$

and $\dfrac{x \quad 3}{2} \quad \dfrac{y}{2} \quad \dfrac{z \quad 2}{1}$

are coplanar. 4

(b) Find the equations of the three planes through the line

$$\frac{x \quad 1}{2} \quad \frac{y \quad 2}{3} \quad \frac{z \quad 3}{4}$$

parallel to the axes. 3

(c) Find the equation of the plane passing through the points $(4, 0, \quad 2)$, $(1, 1, \quad 1)$ and origin. 3

8. *(a)* Find the equations of the planes bisecting the angles between the planes

$$x \quad 2y \quad 2z \quad 3 \quad 0$$

$$3x \quad 4y \quad 12z \quad 1 \quad 0$$ 4

(b) Prove that the equation of the line through the point (, ,) perpendicular to the plane $ax \quad by \quad cz \quad d \quad 0$ is

$$\frac{x}{a} \quad \frac{y}{b} \quad \frac{z}{c}$$

and find the perpendicular distance of the point (, ,) from the plane $ax \quad by \quad cz \quad d \quad 0$. 6

UNIT—V

9. (a) Find the radius and centre of the circle where the plane $x \quad 2y \quad 2z \quad 3$ intersects the sphere

$$x^2 \quad y^2 \quad z^2 \quad 8x \quad 4y \quad 8z \quad 45$$ 5

(b) Find the equations of the tangent planes to the sphere

$$x^2 \quad y^2 \quad z^2 \quad 2x \quad 4y \quad 6z \quad 2 \quad 0$$

parallel to the plane $x \quad y \quad z \quad 0$. 5

10. (a) Find the equation of the cylinder generated by the lines parallel to the line $\frac{x}{1} \; \frac{y}{2} \; \frac{z}{1}$ and intersecting the guiding curve $z \quad 3, \; x^2 \quad y^2 \quad 4$. 5

(b) Prove that the locus of the point from which three mutually perpendicular lines can be drawn to intersect the conic $z \quad 0, \quad ax^2 \quad by^2 \quad 1$ is given by $ax^2 \quad by^2 \quad (a \quad b)z^2 \quad 1$. 5

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Subject Code : MATH/IV/04

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 4th Semester
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Subject ....
Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

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DEGREE 4th Semester
(Arts / Science / Commerce /
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Roll No. ....
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Subject ....
Paper ....

Descriptive Type
Booklet No. B ....

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MATH/IV/04

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(4th Semester)

MATHEMATICS

Paper : MATH–241

(Vector Calculus and Solid Geometry)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—I

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. If $\vec{a}$ and $\vec{b}$ are non-zero vectors and $|\vec{a} \quad \vec{b}| \quad |\vec{a} \quad \vec{b}|$, then $\vec{a}$ and $\vec{b}$ are

(a) perpendicular to each other ☐

(b) parallel to each other ☐

(c) neither parallel nor perpendicular ☐

(d) None of the above ☐

2. If $\vec{f}(t)$ be a vector valued function with constant direction, then

(a) $\vec{f} \quad \frac{d\vec{f}}{dt} \quad 0$ ☐

(b) $\vec{f} \quad \frac{d\vec{f}}{dt} \quad \vec{0}$ ☐

(c) $\left| \vec{f} \quad \frac{d\vec{f}}{dt} \right| \quad \left| \vec{f} \quad \frac{d\vec{f}}{dt} \right|$ ☐

(d) None of the above ☐

3. If the vector $\vec{V} \quad y^2 z\hat{i} \quad a\,xyz\,\hat{j} \quad xy^2\hat{k}$ be a conservative vector, then a is equal to

(a) 0 ☐

(b) 2 ☐

(c) 1 ☐

(d) None of the above ☐

4. Let C be the unit circle in xy-plane with centre at origin. Then $\oint_C y\,dx \quad x\,dy$ is equal to

(a) 2 ☐

(b) ☐

(c) 0 ☐

(d) None of the above ☐

5. By changing the origin to (2, 3) without changing the direction of axes the equation $5x \quad 3y \quad 3$ changes to

(a) $5x \quad 3y \quad 1$ ☐

(b) $5x \quad 3y \quad 6 \quad 0$ ☐

(c) $5x \quad 3y \quad 16 \quad 0$ ☐

(d) None of the above ☐

6. The equation $14x^2 \quad 4xy \quad 11y^2 \quad 44x \quad 58y \quad 71 \quad 0$ represents

(a) an ellipse ☐

(b) a parabola ☐

(c) a hyperbola ☐

(d) None of the above ☐

7. The angle of inclination of the line $x \quad y \quad 0, z \quad 0$ with z-axis is

(a) 45° ☐

(b) 60° ☐

(c) 90° ☐

(d) None of the above ☐

8. The shortest distance between the line

$$\frac{x \quad 1}{2} \quad \frac{y \quad 2}{3} \quad \frac{z \quad 3}{4}$$

and the x-axis is

(a) $\frac{2}{5}$ ☐

(b) $\frac{1}{5}$ ☐

(c) 5 ☐

(d) None of the above ☐

9. The general equation of the sphere which touches the xy-plane at origin is

(a) $x^2 \quad y^2 \quad z^2 \quad 2ux \quad 0$ ☐

(b) $x^2 \quad y^2 \quad z^2 \quad 2vy \quad 0$ ☐

(c) $x^2 \quad y^2 \quad z^2 \quad 2wz \quad 0$ ☐

(d) None of the above ☐

10. The equation of the cone with its vertex at origin and direction cosines of its generators satisfying the relation $4l^2 \quad 5m^2 \quad 7n^2 \quad 0$ is

(a) $4x^2 \quad 5y^2 \quad 7z^2 \quad 0$ ☐

(b) $4x^2 \quad 5y^2 \quad 7z^2 \quad 0$ ☐

(c) $4x^2 \quad 5y^2 \quad 7z^2 \quad 0$ ☐

(d) None of the above ☐

SECTION—II

(*Marks* : 15)

Each question carries 3 marks

State *True* or *False* by putting a Tick ☑ mark in the box provided and give a brief justification :

1. The set of vectors $\hat{i} \quad \hat{j}$, $\hat{j} \quad \hat{k}$ and $\hat{k} \quad \hat{i}$ has reciprocal set of vectors.

True ☐ *False* ☐

Justification :

2. The vector

$$\vec{V} \quad (\ \ 4x \quad 3y \quad 4z)\hat{i} \quad (\ \ 3x \quad 3y \quad 5z)\hat{j} \quad (4x \quad 5y \quad 3z)\hat{k}$$

is irrotational.

True ☐ *False* ☐

Justification :

3. The pair of straight lines $x^2 \quad 5xy \quad 6y^2 \quad 0$ are perpendicular to the pair of straight lines $6x^2 \quad 5xy \quad y^2 \quad 0$.

True ☐ *False* ☐

Justification :

4. The line

$$\frac{x \quad 1}{2} \quad \frac{y \quad 1}{1} \quad \frac{z}{1}$$

wholly lies on the plane $x \quad 2y \quad z \quad 3$.

True ☐ *False* ☐

Justification :

5. The line joining the centre of the sphere $x^2 \quad y^2 \quad z^2 \quad a^2$ to any point P is perpendicular to the polar plane of P with respect to the sphere $x^2 \quad y^2 \quad z^2 \quad a^2$.

True ☐ *False* ☐

Justification :

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