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(CBCS)

(3rd Semester)

MATHEMATICS

THIRD PAPER

(Differential Equations)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. (a) Solve the differential equation

$$\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$$ 5

(b) Solve the differential equation

$$\frac{dy}{dx}+\frac{y}{x}=y^2$$ 5

2. (a) Solve the differential equation

$$x\frac{dy}{dx}+y\log y=xye^x$$ 5

(b) Solve the differential equation

$$\cos(x+y)\,dy=dx$$ 5

UNIT—II

3. (a) Solve the differential equation

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=x\cos x$$ 5

(b) Solve the differential equation

$$\frac{d^3y}{dx^3}-y=(e^x+1)^2$$ 5

4. (a) Find the solution of the differential equation

$$\frac{d^2y}{dx^2}+4y=8\cos 2x$$

given that $y=0$ and $\frac{dy}{dx}=0$, when $x=0$. 5

(b) Solve the differential equation

$$(D^2+4)y=x\sin^2 x$$

where $D=\frac{d}{dx}$. 5

UNIT—III

5. (a) Solve the differential equation

$$p^3 - p(y+3) + x = 0$$

where $p = \dfrac{dy}{dx}$. 5

(b) Find the orthogonal trajectories of the family of curves

$$x^2 + y^2 + 2gy - 1 = 0$$

g being a parameter. 5

6. (a) Reduce the differential equation $y^2(y - px) = p^2x^4$ into Clairaut's form and solve completely, where $p = \dfrac{dy}{dx}$. 5

(b) Solve $(px - y)(py + x) = h^2 p$ by using the transformation $x^2 = u$ and $y^2 = v$, where $p = \dfrac{dy}{dx}$. 5

UNIT—IV

7. (a) Solve the differential equation

$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x)$$ 5

(b) Solve the simultaneous equations

$$\frac{dx}{dt} + 7x - y = 0, \quad \frac{dy}{dt} + 2x + 5y = 0$$ 5

8. (a) Show that

$$x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$$

is exact. Hence solve it completely. 5

(b) Solve the differential equation

$$x\frac{d^2y}{dx^2} - (2x-1)\frac{dy}{dx} + (x-1)y = 0$$ 5

UNIT—V

9. (a) Solve the partial differential equation

$$(y^3x - 2x^4)p + (2y^4 - x^3y)q = 9z(x^3 - y^3)$$

where p and q have their usual meanings. 5

(b) Find the singular solution of the partial differential equation

$$z = px + qy + c\sqrt{1 + p^2 + q^2}$$

where p and q have their usual meanings. 5

10. (a) Find the integral surface of the equation

$$(x-y)y^2p + (y-x)x^2q = (x^2+y^2)z$$

through the curve $xz = a^3$, $y = 0$. 5

(b) Apply Charpit's method to solve

$$p(1+q^2) - q(z-a) = 0$$

where p and q have their usual meanings. 5

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Subject Code : MATH/III/EC/03

Booklet No. A

To be filled in by the Candidate

CBCS

DEGREE 3rd Semester

(Arts / Science / Commerce /

......................) Exam., **2017**

Subject ....

Paper ....

Date Stamp ....

....

To be filled in by the Candidate

CBCS

DEGREE 3rd Semester

(Arts / Science / Commerce /

......................) Exam., **2017**

Roll No. ....

Regn. No. ....

Subject ....

Paper ....

Descriptive Type

Booklet No. B ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

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2 0 1 7

(CBCS)

(3rd Semester)

MATHEMATICS

THIRD PAPER

(Differential Equations)

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick (✓) mark against the correct answer in the brackets provided :

1. The differential equation of the curve $y = \alpha x + \beta x^2$, where α, β are parameters, is

(a) $x^2 y'' - 2xy' + 2y = 0$ ( )

(b) $y'' - 2\beta = 0$ ( )

(c) $y'' + y' + y = 0$ ( )

(d) $y' - \alpha = 0$ ( )

2. The integrating factor for which the differential equation $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$ will be exact, is

(a) x^2y^2 ( )

(b) $\frac{1}{x^2y^2}$ ( )

(c) $e^{x^2y^2}$ ( )

(d) $\frac{y^2}{x^2}$ ( )

3. The value of $\frac{1}{(D-a)^r}e^{ax}$, where $D = \frac{d}{dx}$, is

(a) $r\,!\,x^r$ ( )

(b) $a^r e^{ax}$ ( )

(c) $\frac{x^r}{r\,!}$ ( )

(d) $x^r e^{ax}$ ( )

4. The particular integral of $(D^2 - 4)y = \cos^2 x$, where $D = \frac{d}{dx}$, is

(a) $-\frac{1}{16}(2 + \cos 2x)$ ( )

(b) $-\frac{1}{8}(1 + \cos^2 x)$ ( )

(c) $x^2 \sin x$ ( )

(d) $-\frac{1}{8}(\cos 2x - 4\sin 2x)$ ( )

5. The solution of $y - 2px = -px^2$, where $p = \frac{dy}{dx}$, is

(a) $y + c = 2\sqrt{cx}$ ( )

(b) $y^2 = 4c(x + c^2)$ ( )

(c) $x^2 + y^2 = c^2$ ( )

(d) $x^2 = cy + c$ ( )

6. The orthogonal trajectories of the curve $y = ax^n$ is

(a) $x^2 + y^2 = c$ ( )

(b) $y = cx$ ( )

(c) $x^2 + ny^2 = c$ ( )

(d) $y^2 = cnx$ ( )

7. The suitable substitution to solve the differential equation

$$(1 + x^2)^2 \frac{d^2y}{dx^2} + 2x(1 + x^2)\frac{dy}{dx} + 4y = 0$$

by changing the independent variable into z is

(a) $z = \frac{2}{1 + x^2}$ ( )

(b) $z = 2(1 + x^2)^{-2}$ ( )

(c) $z = \frac{4}{(1 + x^2)^2}$ ( )

(d) $z = 2\tan^{-1} x$ ( )

8. For the equation $(D^2 + PD + Q)y = 0$, where $D = \dfrac{d}{dx}$ and P, Q are functions of x or constant, which of the following is incorrect?

(a) $y = x$ is a particular solution of $P + xQ = 0$ ( )

(b) $y = e^x$ is a particular solution of $1 + P + Q = 0$ ( )

(c) $y = e^{-x}$ is a particular solution of $1 - P + Q = 0$ ( )

(d) $y = e^{mx}$ is a particular solution of $mP + Q = 0$ ( )

9. The partial differential equation obtained by eliminating the arbitrary function from $z = xy + f(x^2 + y^2)$ is

(a) $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = x + y$ ( )

(b) $\left(\dfrac{\partial z}{\partial x}\right)^2 + \left(\dfrac{\partial z}{\partial y}\right)^2 = 1$ ( )

(c) $y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y} = y^2 - x^2$ ( )

(d) $x\dfrac{\partial z}{\partial y} - y\dfrac{\partial z}{\partial x} = 0$ ( )

10. The singular integral of $z = px + qy + pq$, where p and q have their usual meaning, is

(a) $z = ax + by + ab$ ( )

(b) $z = x + y$ ( )

(c) $z = xy$ ( )

(d) $z = x^2 + y^2$ ( )

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. *(a)* Solve the differential equation $y - x\left(\dfrac{dy}{dx}\right) = x + y\left(\dfrac{dy}{dx}\right)$.

Or

(b) Check the exactness of the differential equation

$$\left\{y\left(1+\frac{1}{x}\right)+\cos y\right\}dx + (x + \log x - x\sin x)dy = 0$$

and solve it.

2. *(a)* Find the particular integral of $(D^3 - D^2 - 6D)y = 1 + x^2$, where $D = \frac{d}{dx}$.

Or

(b) Solve $\frac{d^2y}{dx^2} = e^x \cos x$.

3. *(a)* Define Clairaut's equation and solve it completely.

Or

(b) Solve $\frac{d^3y}{dx^3} - 4xy\frac{dy}{dx} + 8y^2 = 0.$

4. *(a)* Solve the differential equation

$$x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$$

Or

(b) Solve

$$x\frac{d^2y}{dx^2} - \frac{dy}{dx} + 4x^3y = x^5$$

by changing the independent variable.

5. *(a)* Solve the partial differential equation

$$p - 2q = 3x^2 \sin(y + 2x)$$

Or

(b) Find the complete integral of $p^2 = qz$.

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