Subject : Mathematics
Paper Name : Algebra
Paper No: II
Semester : II Semester

A. Multiple choice questions [25 (5 from each unit)]

1. The number of generators of a cyclic group of order 8 is
   a) 2
   b) 7
   c) 4
   d) 8

2. The identity element of the group of all positive rational numbers under the composition $a * b = \frac{ab}{2}$ is
   a) 1
   b) 2
   c) 0
   d) -2

3. In multiplicative group of rational numbers, the order of 2 is
   a) 0
   b) Infinite
   c) 1
   d) -1

4. The number of binary compositions on a finite set A having n elements is
   a) n^{n^2}
   b) 2^{n^2}
   c) n^n
   d) n!

5. The identity element in a group $(Z, \times)$, where Z is a set of integers and $\times$ is an ordinary multiplication, is
   a) 0
   b) 1
   c) -1
   d) None of the above

6. When 45^{16} is divided by 32, then the remainder is
   a) 1
   b) 32
   c) 44
   d) 16

7. A homomorphism $f : G \rightarrow G'$ is said to be an isomorphism, if f is
   a) One-to-one mapping
   b) into mapping
   c) one-to-one and into mapping
   d) one-to-one and onto mapping

8. If f is a homomorphism of G *into* G', then the set K of all those elements of G which are mapped by f onto the identity element of G' is called
   a) Kernel of the homomorphism f
   b) Homomorphism f
   c) Kernel of the isomorphism f
   d) Isomorphism f

9. A homomorphism of a group into itself is called
   a) an isomorphism
   b) kernel of a homomorphism
   c) an endomorphism
   d)an automorphism

10. When 7^{10} is divided by 11, then the remainder is
   a) 7
   b)1
   c) 8
   d) 6

11. If $f(x)$ and $g(x)$ be two polynomials of degrees m and n respectively, then $f(x).g(x)$ is a polynomial of degree
   a) m.n
   b) m+n
   c) m/n
   d) n/m

12. The value of the remainder , when $x^3 + 5x^2 + 1$ is divided by $x + 3$, is
   a) 18
   b) -19
   c) 27
   d) 19

13. The expression $x^5 - 61x + p$ is divided by $(x + 1)$, then the value of p is
   a) 62
   b) 60
   c) -60
   d) 6

14. If $f(x)$ and $g(x)$ are non-zero polynomials in F[x], then $f(x) + g(x)$ is non-zero and $\deg(f(x) + g(x))$ is

a) $\deg(f(x)) + \deg(g(x))$
b) $\max\{\deg(f(x)), \deg(g(x))\}$
c) $\deg\{f(x)\}.\deg\{g(x)\}$
d) $\min\{\deg(f(x)), \deg(g(x))\}$

15. If $f(x)$ is divided by $(ax - b)$, then the remainder is

a) $f\left(-\frac{b}{a}\right)$
b) $f\left(\frac{b}{a}\right)$
c) $f(-a)$
d) $f(a)$

16. If *f(x)* and *g(x)* are non-zero polynomials of degree 3 and 5 respectively. Then the value of *deg(f(x)+g(x))* and *deg(f(x).g(x))* are:

(a) 3 & 5
(b) 3 & 8
(c) 5 & 15
(d) 5 & 8

17. If $f(x) = 3x^2 + 5x - 8$ is divided by $(x+1)$, then the remainder is:

(a) 10
(b) 8
(c) -10
(d) -8

18. The expansion of $x^4 - 4x^3 + 3x^2 + 3x + 7$ on the power of $(x-1)$ is:

(a) $(x-1)^4 - (x-1)^2 + (x-1) + 5$
(b) $(x-1)^3 - 4(x-1)^2 + (x+1) + 10$
(c) $(x-1)^4 - 3(x-1)^2 + 2(x-1) + 10$
(d) $(x-1)^4 - 3(x-1)^2 + (x-1) + 10$

19. Which of the following theorem declare that-
for a polynomial with integer coefficients $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$, if there exist prime number p, such that p divides each a_i, $i \neq n$
p does not divide a_n
p^2 does not divide a_0, then f(x) is irreducible over rationals ?

(a) Eisenstein's Irreducibility Criterion
(b) Unique factorization theorem
(c) Euclidean algorithm
(d) Remainder theorem

20. If f(x) be a polynomial and (x-a) is a factor of f(x) then f(a) is equal to:

(a) 2
(b) 0
(c) a

(d) 1

21. Which of the following statement is false ?
(a) A polynomial of degree *n* has n-roots.
(b) A polynomial of degree *n* has more than n-roots.
(c) Surd roots occur in pairs.
(d) Imaginary roots occur in pairs.

22. The equation $x^{12} - x^4 + x^3 - x^2 + 1 = 0$ has.
(a) 3 real roots and 3 complex roots
(b) At least six complex roots.
(c) 2 real roots and 4 complex roots
(d) At least 6 real roots.

23. If the sum of two roots of the equation $x^3 - 5x^2 - 16x + q = 0$ is zero, then the value of q is:
(a) 90
(b) 80
(c) 70
(d) 60

24. If α, β, γ be the root of the equation $x^3 + x + 1 = 0$, then the value of $\alpha^2 + \beta^2 + \gamma^2$ IS:
(a) -2
(b) 1
(c) 2
(d) 1

25. The De Moivre's form of complex number $3-4i$ is:
(a) $(\cos\theta + i\sin\theta)$
(b) $5(\cos\theta - i\sin\theta)$
(c) $5(\cos\theta + i\sin\theta)$
(d) $(\cos\theta - i\sin\theta)$

B. Fill in the blanks:

1. The union of two subgroups of a group is _____ a subgroup.

2. If every element of a group is its own inverse, the G is _____

3. In the set of integer I, inverse of $a \in I$ with respect to addition is ___

4. If $f : G \to G'$ is a homomorphism and $f(G)$ is the homomorphic image of G in G', then $f(G)$ is _____ of G'

5. Every isomorphic image of a cyclic group is _____

6. Let $f : G \to G'$ be a group of homomorphism. Then $Ker\, f = \{e\}$ if and only if f is an _____

7. If the leading coefficient of a polynomial f(x) is 1, then f(x) is said to be _____

8. A polynomial f(x) is completely divisible by (x-h) if and only if _____

9. A polynomial of degree 2 or 3 is irreducible over the field F if and only if it has _____in F

10. The leading coefficient of a polynomial of degree *n* cannot be equal to___.

11. The value of *k* for which the expression $4x^3 - 3x^2 + 2x + k$ is divisible by x+2 is ___.

12. When $4x^5 + 3x^3 + 6x^2 + 5$ is divided by $2x+1$, the remainder is____.

13. The common root of $x^3 - 2x^2 - x + 2 = 0$ and $x^3 + 3x^2 + 2x = 0$ is____.

14. The range of values of k for which the equation $x^4 + 4x^3 - 8x^2 + k = 0$ has all real roots in___ and ___.

15. If α, β, γ are the roots of the cubic equation $a_0x^3 + a_1x^2 + a_2x + a_3 = 0$, then $\sum\alpha\beta$ is equal to___.

Answer Key:

A. **1. (c) 2. (b) 3. (b) 4. (a) 5. (b) 6. (a) 7. (d) 8. (a) 9. (c) 10. (b) 11. (b) 12. (d) 13. (c) 14. (a) 15. (b) 16.(c) 17.(c) 18.(d) 19.(a) 20.(b) 21.(b) 22. (d) 23.(b) 24.(a) 25.(c)**

B.

1. Not necessarily	**2. Abelian**	**3. –a**	**4. a subgroup**
5. Cyclic	**6. Isomorphism**	**7. Monic**	**8. f(h) = 0**
9. no roots	**10. 0**	**11. 48**	**12. 6**
13. -1	**14. 0 and 3**	**15. a_2/a_0**	