**2 0 1 7**

( CBCS )

( 1st Semester )

MATHEMATICS

FIRST PAPER

**( Calculus )**

*Full Marks* : 75

*Time* : 3 hours

( PART : B—DESCRIPTIVE )

( *Marks* : 50 )

*The figures in the margin indicate full marks for the questions*

Answer **five** questions, selecting **one** from each Unit

UNIT—I

**1.** *(a)* Draw the graph of the function defined by

$$\begin{aligned} f(x) \quad & x \quad\quad, \; 0 \quad x \quad 1 \\ & 2 \quad x \;, \; 1 \quad x \quad 2 \\ & x \quad 2 \;, \; 2 \quad x \quad 3 \end{aligned}$$

Discuss whether $f(x)$ exists at $x \quad 2$. 6





*(b)* Using definition of continuity, prove that $f(x) \quad 2x \quad 5$ is continuous at $x \quad 2$. 4

**2.** *(a)* Use L'Hospital rule to evaluate

$$\lim_{x \quad 0} \frac{e^x \quad e^{\quad x} \quad 2\cos x}{x \sin x}$$ 4

*(b)* If $y \quad \tan^{\quad 1} x$, then prove that

$$(1 \quad x^2) y_{n \quad 1} \quad 2nxy_n \quad n(n \quad 1) y_{n \quad 1} \quad 0$$

and find $(y_n)_0$ at $x \quad 0$. 6

UNIT—II

**3.** *(a)* State and prove Lagrange's mean-value theorem. 1+4=5

*(b)* Prove by Maclaurin's theorem that

$$\log(1 \quad x) \quad x \quad \frac{x^2}{2} \quad \frac{x^3}{3} \quad \frac{x^4}{4} \quad \ldots.$$ 5

**4.** *(a)* If $f(h) \quad f(0) \quad hf\,(0) \quad \frac{h^2}{2!} f\,( \quad h), \; 0 \quad\quad 1$, find $\;$, when $h \quad 1$ and $f(x) \quad (1 \quad x)^{\frac{5}{2}}$. 5

*(b)* Expand $2x^3 \quad 7x^2 \quad x \quad 1$ in powers of $x \quad 2$. 5

## UNIT—III

**5.** *(a)* Evaluate : 5

$$\frac{1}{1 \quad x^3}\,dx$$

*(b)* Use the definition of the definite integral as a limit of sum to evaluate $\,_1^2 \frac{1}{x^2}\,dx$. 5

**6.** *(a)* Prove that

$$\,_0^{\frac{}{2}} \cos^n x\,dx \quad \frac{(n \quad 1)(n \quad 3)(n \quad 5)\ldots 3 \quad 1}{n(n \quad 2)(n \quad 4)\ldots 4 \quad 2} \quad \frac{}{2}, \text{ when } n \text{ is even}$$

$$\frac{(n \quad 1)(n \quad 3)(n \quad 5)\ldots 4 \quad 2}{n(n \quad 2)(n \quad 4)\ldots 5 \quad 3} 1, \text{ when } n \text{ is odd.}$$ 6

*(b)* Evaluate :

$$\,_0^{\frac{}{4}} \tan^6 x\,dx$$ 4

## UNIT—IV

**7.** *(a)* Investigate the continuity at (0, 0) of

$$f(x, y) \quad \begin{matrix} 0 & , & (x, y) \quad (0, 0) \\ \dfrac{x^2 \quad y^2}{x^2 \quad y^2} & , & (x, y) \quad (0, 0) \end{matrix}$$ 5





*(b)* If $V \quad \tan^{\,1} \dfrac{2x \quad 3y}{xy}$, then verify that $\tan V \quad \dfrac{2x \quad 3y}{xy}$ is a homogenous function of $x$ and $y$ of degree –1. Prove that

$$x\frac{V}{x} \quad y\frac{V}{y} \quad \frac{1}{2}\sin 2V \quad 0$$ 5

**8.** *(a)* Draw a rough sketch and find the area of the region bounded by the parabolas $y^2 \quad 4x$ and $x^2 \quad 4y$, using the method of integration. 5

*(b)* Evaluate

$$[2a^2 \quad 2a(x \quad y) \quad (x^2 \quad y^2)]\,dx\,dy$$

the region of integration being the circle $x^2 \quad y^2 \quad 2a(x \quad y) \quad 2a^2$. 5

## UNIT—V

**9.** *(a)* Prove that a convergent sequence is bounded. 5

*(b)* Prove that the sequence $\{u_n\}$, where $u_n \quad \dfrac{3n \quad 1}{n \quad 2}$ is monotonic increasing and bounded. Also find its limit. 5

**10.** *(a)* Define Cauchy sequence and prove that every convergent sequence is Cauchy sequence. 5

*(b)* Test the convergent of the series

$$_{n \quad 0} \frac{n}{n \quad 1}^{\;2}$$ 5

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**Subject Code : MATH/I/EC/01**

**Booklet No. A**

Date Stamp ............................

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**To be filled in by the Candidate**

CBCS

DEGREE 1st Semester

(Arts / Science / Commerce / ....................... ) Exam., **2017**

Subject ......................................

Paper .........................................

**To be filled in by the Candidate**

CBCS

DEGREE 1st Semester

(Arts / Science / Commerce / ....................... ) Exam., **2017**

Roll No. .................................

Regn. No. ..............................

Subject ..................................

Paper .....................................

Descriptive Type

Booklet No. B .........................

**INSTRUCTIONS TO CANDIDATES**

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

*Signature of Scrutiniser(s)*

*Signature of Examiner(s)*

*Signature of Invigilator(s)*

**/19**

**MATH/I/EC/01**

**2 0 1 7**

( CBCS )

( 1st Semester )

# MATHEMATICS

FIRST PAPER

**( Calculus )**

( PART : A—OBJECTIVE )

( *Marks* : 25 )

## SECTION—A

( *Marks* : 10 )

*Each question carries 1 mark*

Put a Tick ☑ mark against the correct answer in the box provided :

**1.** $\lim_{x \to 1} \frac{\log x}{x - 1}$ is equal to

(a) 0 ☐

(b) 1 ☐

(c) $e$ ☐

(d) $-1$ ☐

**2.** If $y \quad Ae^{2x} \quad Be^{\ 2x}$, then $\dfrac{d^2y}{dx^2}$ is

*(a)* $y$ ☐

*(b)* $x \quad y$ ☐

*(c)* $4y$ ☐

*(d)* None of the above ☐

**3.** $f(x) \quad \log x$ can be expanded in power of $(x \quad 1)$ by using

*(a)* Maclaurin's theorem ☐

*(b)* Taylor's theorem ☐

*(c)* Leibnitz's theorem ☐

*(d)* None of the above ☐

**4.** Using mean-value theorem, the point to the curve $y \quad x^2$ where the tangent is parallel to the line joining the points (1, 1) and (2, 4) is

*(a)* (1, 9/4) ☐

*(b)* (3/4, 9/4) ☐

*(c)* (3/2, 9/4) ☐

*(d)* None of the above ☐

**5.** If for an even function $f(x)$, $\,_{0}^{1} f(x)dx \quad 4$, then the value of $\,_{1}^{1} f(x)dx$ is

*(a)* 4 ☐

*(b)* 4 ☐

*(c)* 8 ☐

*(d)* 8 ☐

**6.** The value of $\,_{0}^{\overline{2}} \cos^6 x \, dx$ is

*(a)* $\frac{}{32}$ ☐

*(b)* $\frac{3}{32}$ ☐

*(c)* $\frac{5}{32}$ ☐

*(d)* None of the above ☐

**7.** The value of $\frac{u}{x}$ when $u \quad \tan^{1}(y / x)$ is

(a) $\frac{x}{x^2 \quad y^2}$ ☐

(b) $\frac{x}{x^2 \quad y^2}$ ☐

(c) $\frac{y}{x^2 \quad y^2}$ ☐

(d) $\frac{y}{x^2 \quad y^2}$ ☐

**8.** The value of

$$\int_{y \; 0}^{y \; \overline{2}} \int_{x \; 0}^{x} \cos(x \quad y)\, dx\, dy$$

is

(a) 2 ☐

(b) 1 ☐

(c) 1 ☐

(d) 2 ☐

**9.** $\underset{n\quad 1}{}\frac{1}{n^{p}}$ is convergent, if

*(a)* $p \quad 1$ ☐

*(b)* $p \quad 1$ ☐

*(c)* $p \quad 1$ ☐

*(d)* None of the above ☐

**10.** The series $\underset{n\quad 1}{a_n}$ of positive terms is divergent according as $\lim\limits_{n} \frac{a_{n\quad 1}}{a_n}$ is

*(a)* equal to 1 ☐

*(b)* greater than 1 ☐

*(c)* less than 1 ☐

*(d)* None of the above ☐

## Section—B

( *Marks* : 15 )

*Each question carries 3 marks*

**1.** *(a)* Find the $n$th derivative of $y \quad \sin(ax \quad b)$.

*Or*

*(b)* Evaluate

$$\lim_{x \quad 0} \frac{\tan x \quad x}{x \quad \sin x}$$

**2.** *(a)* Expand $\sin x$ by Maclaurin's theorem.

*Or*

*(b)* Verify Lagrange's mean-value theorem for $f(x) \quad x^2 \quad 2x \quad 3$ on $[4, 6]$.

**3.** *(a)* Evaluate

$$\log x \, dx$$

*Or*

*(b)* Evaluate

$${}_{0}^{\overline{2}} \; \frac{\sin^3 x}{\sin^3 x \quad \cos^3 x} \, dx$$

**4.** *(a)* Find $\frac{{}^2u}{x^2}$, when $u \quad \log(x^2 \quad y^2)$.

*Or*

*(b)* If $u \quad x^2y \quad y^2z \quad z^2x$, then show that

$$\frac{u}{x} \quad \frac{u}{y} \quad \frac{u}{z} \quad (x \quad y \quad z)^2$$

**5.** *(a)* If $\{a_n\}$ and $\{b_n\}$ are convergent sequences, then prove that $\{a_n \quad b_n\}$ is a convergent sequence.

*Or*

*(b)* Prove that the series

$$_{n \quad 1} \frac{n}{2n \quad 5}$$

is convergent.

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