V/MAT (viii) (B)

2 0 1 6

(5th Semester)

MATHEMATICS

Paper : MATH–354(B)

(Probability Theory)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* State the axiomatic definition of probability. For any two events A and B, prove that

$$P(\bar{A} \quad B) \quad P(B) \quad P(A \quad B)$$ 5

(b) From a city population, the probability of selecting

(i) a male or a smoker is 7/10

(ii) a male smoker is 2/5

(iii) A male, if a smoker is already selected is 2/3

Find the probability of selecting

(1) a non-smoker;

(2) a male;

(3) a smoker, if a male is first selected. 5

2. *(a)* State and prove Bayes' theorem. 7

(b) A factory produces a certain type of outputs by three types of machine. The respective daily production figures are :

Machine I : 3000 units

Machine II : 2500 units

Machine III : 4500 units

Past experience shows that 1 percent of the output produced by Machine I is defective. The corresponding fraction of defectives for the other two machines are 1·2 percent and 2 percent respectively. An item is drawn at random from the day's production run and is found to be defective. What is the probability that it comes from the output of Machine I ? 3

UNIT—II

3. *(a)* A continuous random variable X has probability distribution function $f(x) \quad 3x^2, \ 0 \quad x \quad 1$. Find a and b such that

(i) $P(X \quad a) \quad P(X \quad a)$

(ii) $P(X \quad b) \quad 0\ 05$ 5

(b) Ten coins are thrown simultaneously. Find the probability of getting at least 7 heads. 5

4. A random variable X is distributed at random between the values 0 and 4 and its probability density function is given by

$$f(x) \quad kx^3 (4 \quad x)^2$$

Find the value of k, the mean, the variance and the standard deviation. 5+3+2=10

UNIT—III

5. For the joint probability distribution of two random variables X and Y given below : 5+5=10

Y / X	1	2	3	4	Total
1	4/36	3/36	2/36	1/36	10/36
2	1/36	3/36	3/36	2/36	9/36
3	5/36	1/36	1/36	1/36	8/36
4	1/36	2/36	1/36	5/36	9/36
Total	11/36	9/36	7/36	9/36	1

(a) Find the marginal distribution of X and Y.

(b) Find the conditional distribution of X, given the value of $Y \quad 1$ and that of Y given the value of $X \quad 2$.

6. Suppose that a two-dimensional continuous random variable (X, Y) has joint probability density function given by : 5+5=10

$$f(x, y) \quad \begin{matrix} 6x^2y, & 0 \quad x \quad 1, \ 0 \quad y \quad 1 \\ 0 \quad , & \text{elsewhere} \end{matrix}$$

(a) Verify that $\int_0^1 \int_0^1 f(x, y)\, dx dy \quad 1$.

(b) Find—

(i) $P \ 0 \quad X \quad \frac{3}{4}, \frac{1}{3} \quad Y \quad 2$

(ii) $P(X \quad Y \quad 1)$

(iii) $P(X \quad Y)$

(iv) $P(X \quad 1 | Y \quad 2)$

UNIT—IV

7. State and prove Chebyshev's inequality. 2+8=10

8. Calculate the correlation coefficients for the following heights (in inches) of fathers (X) and their sons (Y) : 10

X	:	65	66	67	67	68	69	70	72
Y	:	67	68	65	68	72	72	69	71

UNIT—V

9. *(a)* For a Poisson distribution, prove that

$$_{r \ 1} \quad r \quad _{r \ 1} \quad \frac{d \ _r}{d} \qquad 7$$

(b) X is a normal variate with mean 30 and standard deviation 5. Find the probability that $X \quad 45$. 3

10. *(a)* If X and Y are independent Poisson variates, show that the conditional distribution of X given $X \quad Y$, is binomial. 7

(b) Find the moment-generating function of the gamma distribution about origin. 3

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Subject Code : V/MAT (viii) (B)

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce /
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Subject ....

Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

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DEGREE 5th Semester
(Arts / Science / Commerce /
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Roll No. ....

Regn. No. ....

Subject ....

Paper ....

Descriptive Type
Booklet No. B ....

Signature of Scrutiniser(s)

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2 0 1 6

(5th Semester)

MATHEMATICS

Paper : MATH–354(B)

(Probability Theory)

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A (Multiple choice)

(*Marks* : 10)

Each question carries 1 mark

Answer **all** questions

Put a Tick ☑ mark against the correct answer in the box provided : 1×10=10

1. A letter of the English alphabet is chosen at random. The probability that the letter so chosen precedes *m* and is a vowel is

(a) $\frac{5}{26}$ ☐

(b) $\frac{3}{26}$ ☐

(c) $\frac{12}{26}$ ☐

(d) None of the above ☐

2. If $P(A \quad B) \quad P(A) \quad P(B)$, then the two events A and B are

(a) mutually exclusive ☐

(b) independent ☐

(c) dependent ☐

(d) None of the above ☐

3. For the binomial distribution

(a) mean < variance ☐

(b) variance < mean ☐

(c) mean = variance ☐

(d) None of the above ☐

4. The parameters of a binomial distribution with mean 4 and variance 3 are

(a) $n \quad 16, \ p \quad \dfrac{1}{2}$ ☐

(b) $n \quad 16, \ p \quad \dfrac{1}{4}$ ☐

(c) $n \quad 32, \ p \quad \dfrac{1}{2}$ ☐

(d) $n \quad 32, \ p \quad \dfrac{1}{4}$ ☐

5. Two random variables X and Y with joint p.d.f. (p.m.f.) $f_{XY}(x, y)$ and marginal p.d.f.'s (p.m.f.'s) $f_X(x)$ and $g_Y(y)$ respectively, are said to be stochastically independent if and only if

(a) $f_{XY}(x, y) \quad f_X(x) \, g_Y(y)$ ☐

(b) $f_{XY}(x, y) \quad f_X(x) / g_Y(y)$ ☐

(c) $f_{XY}(x, y) \quad f_X(x) \quad g_Y(y)$ ☐

(d) $f_{XY}(x, y) \quad f_X(x) \quad g_Y(y)$ ☐

6. The conditional probability density function of Y given X for two random variables X and Y which are jointly continuously distributed is given by

(a) $f_{X|Y}(x|y) \quad \frac{}{x} F_{Y|X}(y|x)$ ☐

(b) $f_{X|Y}(x|y) \quad \frac{}{y} F_{X|Y}(y|x)$ ☐

(c) $f_{Y|X}(y|x) \quad \frac{}{x} F_{Y|X}(y|x)$ ☐

(d) $f_{Y|X}(y|x) \quad \frac{}{y} F_{Y|X}(y|x)$ ☐

7. For two random variables X and Y, $\text{var}(X \quad Y)$ is equal to

(a) $\text{var}(X) \quad \text{var}(Y)$ ☐

(b) $\text{var}(X) \quad \text{var}(Y)$ ☐

(c) $\text{var}(X) \quad \text{var}(Y) \quad 2\,\text{cov}(X, Y)$ ☐

(d) $\text{var}(X) \quad \text{var}(Y) \quad 2\,\text{cov}(X, Y)$ ☐

8. For two random variables X and Y, the relation $E(XY) \quad E(X)\ E(Y)$ holds good

(a) if X and Y are independent ☐

(b) if X and Y are identical ☐

(c) for all X and Y ☐

(d) None of the above ☐

9. The mean of a geometric distribution is

(a) pq ☐

(b) p/q ☐

(c) q/p ☐

(d) None of the above ☐

10. For exponential distribution, when $\quad 1$

(a) variance < mean ☐

(b) variance = mean ☐

(c) variance > mean ☐

(d) None of the above ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

Answer **all** questions

1. If A and B are mutually independent events, then A^c and B are also mutually independent. State true or false and justify your answer.

2. If X is uniformly distributed over the interval $[a, b]$, prove that

$$E(X) \quad \frac{a \quad b}{2}$$

3. If X and Y are random variables having joint density function

$$f\,(x,\ y) \quad \begin{array}{l} \frac{1}{8}(6 \quad x \quad y), \quad 0 \quad x \quad 2, \quad 2 \quad y \quad 4 \\ \qquad 0 \qquad\quad , \quad \text{otherwise} \end{array}$$

find $P\,(X \quad 1 \quad Y \quad 3)$.

4. For random variables X and Y, prove that $E(X \quad Y) \quad E(X) \quad E(Y)$, provided all the expectations exist.

5. If X is a Poisson variate such that

$$P(X \quad 2) \quad 9P(X \quad 4) \quad 90P(X \quad 6)$$

find the mean.

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