V / MAT (vii)

2014

(5th Semester)

MATHEMATICS

Paper : MATH–353

(Complex Analysis)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Show that two points z_1, z_2 will be inverse points w.r.t. the circle

$$z\bar{z}+\bar{\alpha}z+\alpha\bar{z}+r=0$$

if and only if

$$z_1\bar{z}_2+\bar{\alpha}z_1+\alpha\bar{z}_2+r=0$$ 5

(b) Prove that

$$|z_1+z_2|^2+|z_1-z_2|^2=2|z_1|^2+2|z_2|^2$$ 5

2. (a) Prove that the sum and product of two complex numbers are real iff they are conjugate to each other. 5

(b) Find the loci of z which satisfy the condition

$$\arg \frac{z-1}{z+1} = \frac{\pi}{3}$$ 5

UNIT—II

3. (a) If n is real, show that

$$r^n (\cos n\theta + i \sin n\theta)$$

is analytic except when $r = 0$ and find its derivatives. 5

(b) Using Cauchy-Riemann equations, show that $f(z) = \bar{z}$ is nowhere differentiable. 5

4. (a) Show that the function

$$f(z) = e^{-z^{-4}}, \; z \neq 0, \; f(0) = 0$$

is not analytic at $z = 0$, although Cauchy-Riemann equations are satisfied at that point. 5

(b) If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, then find $f(z)$ in term of z. 5

UNIT—III

5. (a) Show that the power series $\sum a_n z^n$ and its derivative $\sum n a_n z^{n-1}$ have same radius of convergence. 4

(b) Find the radius of convergence of the following power series : 6

(i) $\sum \frac{n\sqrt{2}+i}{1+2ni} z^n$

(ii) $\frac{z}{2} + \frac{1.3}{2.5} z^2 + \frac{1.3.5}{2.5.8} z^3 + \cdots$

6. (a) Find the domain of convergence of the series

$$\sum \left(\frac{iz-1}{2+i} \right)^n$$ 5

(b) Examine the behaviour of the following power series on the circle of convergence : 5

(i) $\sum_{1}^{\infty} \frac{z^n}{n}$

(ii) $\sum_{2}^{\infty} \frac{z^{4n}}{4n+1}$

UNIT—IV

7. (a) Verify Cauchy integral theorem for the following integral : 5

$$\oint_C (2z^2 - iz + 3)\, dz, \quad C: |z| = 1$$

(b) Evaluate

$$\int_L \frac{z+2}{z}\, dz$$

where L is the semicircle $|z| = 2$. 5

8. (a) Using Cauchy integral formula, evaluate

$$\oint_C \frac{dz}{z(z^2+4)}, \quad C: |z| = 1$$ 5

(b) Expand $\dfrac{1}{z(z^2 - 3z + 2)}$ for the region $1 < |z| < 2$. 5

UNIT—V

9. (a) Show that the function $\dfrac{1}{z^2 - 1}$ has simple poles at $z = 1$ and $z = -1$. 5

(b) Show that $\exp\left[\frac{c}{2}(z - z^{-1})\right]$ can be expanded in the form $\sum_{-\infty}^{\infty} a_n z^n$ and find the value of a_n. 5

10. *(a)* Find the singularity of the following functions : 4

(i) $\dfrac{\cot \pi z}{(z-a)^2}$ at $z = a$ and $z = \infty$

(ii) $\sin \dfrac{1}{1-z}$ at $z = 1$

(b) State and prove maximum modulus theorem. 6

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2014

(5th Semester)

MATHEMATICS

Paper : MATH–353

(Complex Analysis)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(Multiple Choice)

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. If $z = 1 + i\sqrt{3}$, then $|\arg z| + |\arg \bar{z}|$ is equal to

   (a) $\pi / 3$ ☐

   (b) $2\pi / 3$ ☐

   (c) 0 ☐

   (d) $\pi / 2$ ☐

2. The conjugate of i is

(a) i ☐

(b) $-i$ ☐

(c) 1 ☐

(d) -1 ☐

3. Functions satisfying Laplace's equation are known as

(a) regular ☐

(b) harmonic ☐

(c) homomorphic ☐

(d) conjugate ☐

4. The analytic function whose real part is $e^x \cos y$ is

(a) e^z ☐

(b) e^{-z} ☐

(c) z ☐

(d) z^2 ☐

5. A power series within its circle of convergence

(a) converges absolutely ☐

(b) converges uniformly ☐

(c) converges absolutely and uniformly ☐

(d) diverges ☐

6. The power series $\sum \frac{1}{z^n}$ will converge

(a) if $|z|<1$ ☐

(b) if $|z|>1$ ☐

(c) if $|z|=1$ ☐

(d) for all real values of z ☐

7. The value of $\int_C \frac{dz}{z}$ along the semicircular arc above to the circle $|z|=1$ is

(a) πi ☐ (b) $-\pi i$ ☐

(c) $2\pi i$ ☐ (d) $-2\pi i$ ☐

8. A continuous arc without multiple points is called a

(a) Jordan curve ☐

(b) continuous arc ☐

(c) contour ☐

(d) rectifiable arc ☐

9. The function $e^{1/z}$ has singularity $z=0$ as a/an

(a) removable singularity ☐

(b) pole ☐

(c) essential singularity ☐

(d) non-isolated essential singularity ☐

10. A function $w=f(z)$ ceases to be analytic, if

(a) $\frac{dw}{dz}=0$ ☐

(b) $\frac{dw}{dz}=\infty$ ☐

(c) $\frac{dw}{dz}$ does not exist ☐

(d) None of the above ☐

SECTION—B

(Very short answer)

(*Marks* : 15)

Each question carries 3 marks

Answer **all** questions

Write very short answer of the following :

1. If $a^2 + b^2 = 1$, then find the value of

$$\frac{1+a+ib}{1+a-ib}$$

2. For what value of z, the function w defined by the equations ceases to be analytic?

$$z = \log \rho + i\phi \, ; \quad w = \rho(\cos\phi + i\sin\phi)$$

3. Find the centre and radius of convergence of the power series

$$\sum \frac{(-1)^n}{n}(z+2i)^n$$

4. Evaluate :

$$\int_{-2+i}^{5+3i} z^3 \, dz$$

5. Define pole with example.

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