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(5th Semester)

MATHEMATICS

Paper : MATH–352

(Real Analysis)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Prove that every open cover of a compact set admits of finite subcover. 6

(b) Show that a set is closed if and only if its complement is open. 4

2. State and prove the Bolzano-Weierrstrass theorem for the subsets of R^n. 2+8=10

UNIT—II

3. *(a)* State and prove intermediate value theorem of a real valued function of several real variables. 1+5=6

(b) Prove that the function

$$\frac{1}{1!} \quad \frac{1}{2!} \quad \ldots \quad \frac{1}{n!}, \quad n \quad N$$

is not continuous at (0, 0). 4

4. *(a)* Define limits of functions of several variables. Let

$$\lim_{x \quad a} f(x) \quad b \text{ and } b \quad (b_1, b_2, \ldots, b_n),$$
$$f \quad (f_1, f_2, \ldots, f_m)$$

then show that $\lim_{x \quad a} f_i(x) \quad b_i, \ 1 \quad i \quad m.$ 2+5=7

(b) Show that the function

$$f(x, y) \quad \frac{x^2 \quad y^2}{x^2 \quad y^2}$$

has repeated limits. 3

UNIT—III

5. *(a)* Given

$$f(x, y) \quad \begin{cases} \dfrac{xy}{\sqrt{x^2 \quad y^2}}, & (x, y) \quad (0, 0) \\ 0, & (x, y) \quad (0, 0) \end{cases}$$

Show that f is continuous, possesses partial derivatives but is not differentiable at (0, 0). 6

(b) Let

$$f(x, y) \quad \begin{cases} \dfrac{x^2 y}{x^4 \quad y^2}, & (x, y) \quad (0, 0) \\ 0, & (x, y) \quad (0, 0) \end{cases}$$

Show that f has a directional derivative at (0, 0) in any direction $\quad (l, m)$, $l^2 \quad m^2 \quad 1$ but f is discontinuous at (0, 0). 4

6. *(a)* Prove that a function which is differentiable at a point admits of partial derivatives at the point. 5

(b) Show that the functions $u \quad x \quad y \quad z$, $v \quad x \quad y \quad z$, $w \quad x^2 \quad y^2 \quad z^2 \quad 2yz$ are not independent. Find the relation among them. 5

UNIT—IV

7. State and prove Young's theorem. 2+8=10

8. *(a)* If

$$f(x, y) \quad \begin{cases} (x^2 \quad y^2)\tan^{\ 1} \dfrac{y}{x}, & x \quad 0 \\ \dfrac{}{2} y^2, & x \quad 0 \end{cases}$$

then show that $f_{xy}(0, 0) \quad f_{yx}(0, 0)$. 5

(b) Show that the function

$$f(x, y) \quad 2x^4 \quad 3x^2 y \quad y^2$$

has neither a maxima nor a minima at (0, 0). 5

UNIT—V

9. *(a)* Let X be the set of all sequences of complex numbers. Show that the function d defined by

$$d(x, y) \quad \frac{1}{2^n} \frac{|x_n \quad y_n|}{(1 \quad |x_n \quad y_n|)},$$
$$x \quad \{x_n\},\ y \quad \{y_n\} \quad X$$

is a metric space. 5

(b) Prove that every compact subset A of a metric space (X, d) is bounded. 5

10. Define complete metric space. Let X be the set of all continuous real valued functions defined on $[0, 1]$ and let

$$d(x, y) = \int_0^1 |x(t) - y(t)|\, dt, \quad x, y \in X$$

Show that (X, d) is not complete. 2+8=10

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Subject Code : MATH/V/06

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce / ....) Exam., **2017**

Subject ....
Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce / ....) Exam., **2017**

Roll No. ....
Regn. No. ....

Subject ....
Paper ....

Descriptive Type
Booklet No. B ....

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

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MATH/V/06

2 0 1 7

(5th Semester)

MATHEMATICS

Paper : MATH–352

(Real Analysis)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. In R^2, the limit point of the set $Q^2 \quad \{(x, y) : x \quad Q, \; y \quad Q\}$ is

(a) (0, 0) ☐

(b) (1, 1) ☐

(c) every point of R^2 ☐

(d) None of the above ☐

2. If every limit point of a set belongs to the set, then the set is

(a) closed ☐

(b) open ☐

(c) derived ☐

(d) None of the above ☐

3. A function $f(x, y)$ is said to be continuous if

(a) it is continuous at isolated point only ☐

(b) it is continuous at each point of its domain ☐

(c) it is continuous at some deleted neighbourhood of domain ☐

(d) None of the above ☐

4. The range of a function continuous on a compact set is

(a) cover ☐

(b) subcover ☐

(c) compact ☐

(d) None of the above ☐

5. If a real valued function f defined in an open set R^n possesses first order partial derivatives at each point of D and the functions $D_1 f$, $D_2 f$, ... $D_n f$ are all continuous, then f is

(a) derivable in ☐

(b) partially derivable in ☐

(c) continuously derivable in ☐

(d) not derivable in ☐

6. $\lim\limits_{h \quad 0} \frac{f(x \quad h, y) \quad f(x, y)}{h}$, if exist is called the partial derivative of f with respect to

(a) x at (a, b) ☐

(b) x at (x, y) ☐

(c) y at (a, b) ☐

(d) y at (x, y) ☐

7. If (a, b) be a point of the domain contained in R^2 of a function f such that f_x and f_y are both differentiable at (a, b), then

(a) $f_{xy}(a, b) \quad f_{yx}(a, b)$ ☐

(b) $f_{xy}(a, b) \quad f_{yx}(a, b)$ ☐

(c) f_{xy} and f_{yx} do not exist at (a, b) ☐

(d) None of the above ☐

8. When

$$f_{xx}(a, b)\ f_{yy}(a, b)\quad \{f_{xy}(a, b)\}^2$$
$$0,\ f_{xx}(a, b)\quad 0,$$
$$f_{yy}(a, b)\quad 0$$

then f is

(a) minimum at (a, b) ☐

(b) maximum or minimum at (a, b) ☐

(c) maximum at (a, b) ☐

(d) None of the above ☐

9. The subset $S\quad \{(x, y): x^2\quad y^2\quad 1,\ \ x, y\quad R\}$ of R^2 with the Euclidean metric is

(a) open ☐

(b) closed ☐

(c) bounded ☐

(d) compact ☐

10. If every Cauchy sequence of X converges to a point of X, then a metric space (X, d) is

(a) compact ☐

(b) interior ☐

(c) complete ☐

(d) closure ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

Answer the following :

1. Prove that the union of a finite number of open sets in R is an open set.

2. Define convex set and uniform continuity.

3. If

$$u_1 \quad \frac{x_2 x_3}{x_1}, \;\; u_2 \quad \frac{x_1 x_3}{x_2}, \;\; u_3 \quad \frac{x_1 x_2}{x_3}$$

then prove that $J(u_1, u_2, u_3) \quad 4$.

4. State Taylor's theorem for a real valued function in R^n.

5. Let A be any subset of a metric space (X, d). Then prove that $A \quad \overline{A}$ if and only if A is closed. ($\overline{A}$ is closure of A)

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