2015

(2nd Semester)

BACHELOR OF COMPUTER APPLICATIONS

Paper No. : BCA–202

[Mathematics—II (Discrete Mathematics)]

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* In a survey of 60 people, it was found that 25 people read newspaper *A*, 26 read newspaper *B*, 26 read newspaper *C*, 9 read both *A* and *C*, 11 read both *A* and *B*, 8 read both *B* and *C*, and 3 read all the three newspapers. Find—

(i) the number of people who read at least one of the newspapers;

(ii) the number of people who read exactly one newspaper. 5

(b) Define a finite Boolean algebra and show that D_{70} is a finite Boolean algebra with the partial order of divisibility, where D_n is defined as the set of all positive divisors of n. 5

2. *(a)* In a survey of 100 students, the number of students studying the various languages is found as English only 18; English but not Hindi 23; English and Sanskrit 8; Sanskrit and Hindi 8; English 26; Sanskrit 48 and no language 24. Find—

(i) how many students are studying Hindi;

(ii) how many students are studying English and Hindi both. 5

(b) Let $S = \{a, b, c\}$ and $L = P(S)$. Prove that $(L, \subseteq)$ is isomorphic to D_{42}. Here $P(S)$ is defined as the power set of S and D_n the set of all positive divisors of n. 5

UNIT—II

3. *(a)* Without constructing the truth table, show the following equivalence

$$(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \equiv r$$ 5

(b) Obtain the principal disjunctive normal form of the formula

$$(p \to (q \wedge r)) \wedge (\neg p \to (\neg q \wedge \neg r))$$ 5

4. *(a)* Without constructing the truth table, show the following equivalence : 5

$$(p \vee q) \wedge (\neg p \wedge (\neg p \wedge q)) \equiv (\neg p \wedge q)$$

(b) Obtain the principal conjunctive normal form of the formula

$$p \vee (\neg p \rightarrow (q \vee (\neg q \rightarrow r)))$$ 5

UNIT—III

5. *(a)* A committee of 5 is to be selected from among 6 boys and 5 girls. Determine the number of ways of selecting the committee if it is to consist of at least one boy and one girl. 5

(b) If the 4th term in the expansion of

$$\left(ax + \frac{1}{x}\right)^n$$

is $\frac{5}{2}$, find the values of n and a. 5

6. *(a)* In an examination paper, there are two groups each containing 7 questions. A candidate is required to attempt 9 questions but not more than 5 questions from each group. In how many ways can 9 questions be selected? 5

(b) If x^p occurs in the expansion of

$$\left(x^2 + \frac{1}{x}\right)^{2n}$$

prove that its coefficient is

$$\frac{(2n)!}{\left(\frac{4n-p}{3}\right)!\left(\frac{2n+p}{3}\right)!}$$ 5

UNIT—IV

7. (a) Show that the set $\mathbb{Z}$ of all integers is an Abelian group with $*$ defined by

$$a * b = a + b + 2$$ 5

(b) Define normal subgroup. Suppose that N and M are two normal subgroups of G and that $N \cap M = \{e\}$. Show that every element of N commutes with every of M. 5

8. (a) Show that the set of all matrices

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

a and b being non-zero reals, is a group under matrix multiplication. 5

(b) Define group homomorphism. Let f be a homomorphic mapping of a group G into a group J. Let $f(G)$ be the homomorphic image of G in J. Then show that $f(G)$ is a subgroup of J. 5

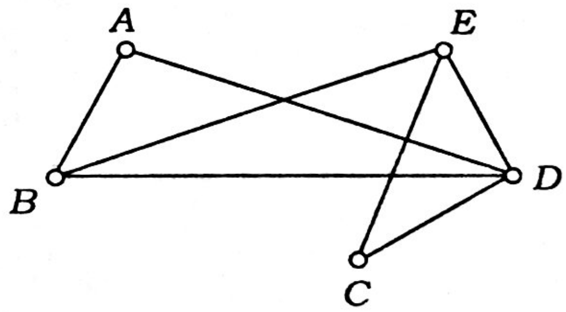
UNIT—V

9. (a) Define the following terms with examples : 5

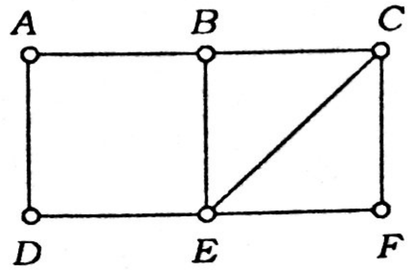
(i) Complete graph

(ii) Planar and non-planar graphs

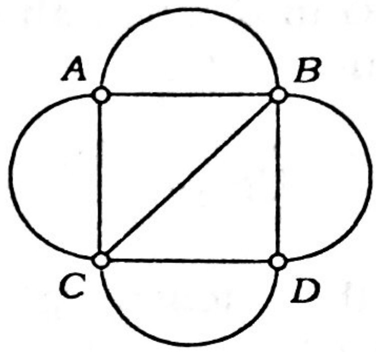
(b) Write the adjacency matrix and the incidence matrix for the following graph : 5



10. (a) Define tree. Write down all the spanning trees of the following graph : 5



(b) Define Hamiltonian graph. Check whether the following graph has Hamiltonian circuit or not :



Give sufficient reason to support your answer. 5

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2015

(2nd Semester)

BACHELOR OF COMPUTER APPLICATIONS

Paper No. : BCA–202

[Mathematics—II (Discrete Mathematics)]

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 15)

I. Put a Tick (✓) mark against the correct answer in the brackets provided : 1×10=10

1. If $A = \{5, 7, 9, 11, 13, 15\}$ and $B = \{x : x = 2n + 1, 2 \leq n \leq 7, n \in \mathbb{N}\}$, then $B - A$ is equal to

   (a) $\{9, 11, 13, 15\}$ ( )

   (b) ϕ ( )

   (c) $\{5, 7, 9\}$ ( )

   (d) B ( )

2. A self-complemented distributive lattice is called

   (a) Boolean algebra ( )

   (b) modular lattice ( )

   (c) complete lattice ( )

   (d) self-dual lattice ( )

3. Let p denote "He is rich" and let q denote "He is happy'. Then the statement formula $q \rightarrow \neg p$ is equivalent to

   (a) if he is rich, then he is unhappy ( )

   (b) it is necessary to be poor in order to be happy ( )

   (c) he is neither rich nor happy ( )

   (d) to be poor is to be unhappy ( )

4. Which of the following propositions is a tautology?

(a) $(p \vee q) \rightarrow p$ ( )

(b) $p \vee (q \rightarrow p)$ ( )

(c) $p \vee (p \rightarrow q)$ ( )

(d) $p \rightarrow (q \rightarrow p)$ ( )

5. The number of different permutations of the word 'BANANA' is

(a) 720 ( )

(b) 120 ( )

(c) 60 ( )

(d) 360 ( )

6. The 3rd term in the expansion of $\left(3x - \frac{y^3}{6}\right)^4$ is

(a) $\frac{2}{3}xy^3$ ( )

(b) $\frac{12}{5}xy^{-2}$ ( )

(c) $-\frac{3}{2}x^3y^5$ ( )

(d) $\frac{3}{2}x^2y^6$ ( )

7. The set of integers $\mathbb{Z}$ with the binary operation $*$ defined as $a * b = a + b + 1$ for $a, b \in \mathbb{Z}$, is a group. The identity element of this group is

(a) -1 ( )

(b) 1 ( )

(c) 0 ( )

(d) 12 ( )

8. A necessary and sufficient condition for a non-empty subset H of a finite group G to be a subgroup is that

(a) $a \in H,\ b \notin H \Rightarrow a,\ b \notin H$ ( )

(b) $a \in H,\ b \in H \Rightarrow (a+b) \in H$ ( )

(c) $a \in H,\ b \in H \Rightarrow ab \in H$ ( )

(d) $a \in H,\ b \in H \Rightarrow (a-b) \in H$ ( )

9. The total number of edges in a complete graph of n vertices is

(a) n ( )

(b) $\frac{n(n-1)}{2}$ ( )

(c) $\frac{n}{2}$ ( )

(d) $n^2 - 1$ ( )

10. If a graph G is bipartite, then the chromatic number (χ) of G is

(a) 1 ( )

(b) 3 ( )

(c) 0 ( )

(d) 2 ( )

II. Tick (✓) either True or False : 1×5=5

11. Let $(P(A), \subseteq)$ be a poset where A is any non-empty finite set, $P(A)$ power set of A and $\subseteq$ is 'set inclusion'. Then the least element of $(P(A), \subseteq)$ is any singleton set.

True ( ) / *False* ( )

12. The contrapositive of the statement 'If $f(2) = 0$, then $f(x)$ is divisible by $(x-2)$' is '$f(x)$ is divisible by $(x-2) \Rightarrow f(2) \neq 0$'.

True ( ) / *False* ( )

13. If ${}^nC_p = {}^nC_q$, then $p = q$ or $p + q = n$.

True ( ) / *False* ( )

14. Burnside's theorem states that if G is a finite group of order $p^a q^b$ where p and q are any integers, and a and b are non-negative integers, then G is solvable.

True ( ) / *False* ( )

15. Sum of the degrees of all regions in a map is equal to twice the number of edges in the corresponding graph.

True ( ) / *False* ( )

SECTION—II

(*Marks* : 10)

III. Answer the following questions : $2\times5=10$

1. Let $\mathbb{N}=\{1, 2, 3, \ldots\}$ be ordered by divisibility. State with brief justification whether each of the following subsets of $\mathbb{N}$ is linearly (totally) ordered :

   (a) $\{24, 2, 6\}$

   (b) $\{3, 15, 5\}$

   (c) $\mathbb{N}=\{1, 2, 3, \ldots\}$

   (d) $\{2, 8, 32, 4\}$

2. Using truth table, prove that the following argument is valid :

$$p \rightarrow \neg q,\ r \rightarrow q,\ r \vdash \neg p$$

3. Find the number of different words beginning with P which can be formed by using all the letters of the word 'PERMUTATION'.

4. Define cosets with an example.

5. Draw a graph having—

 (a) one cutpoint and one bridge;

 (b) no cutpoint and no bridge.

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