2015

(5th Semester)

MATHEMATICS

PAPER : MATH–354(B)

(Probability Theory)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Show that if an event A is independent of the events B, $B \cap C$ and $B \cup C$, then it is also independent of C. 5

(b) Prove that for any two events A and B

$$P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$$ 5

(b) Express

$$3x^4 - 4x^3 + 6x^2 + 2x + 1$$

in terms of a factorial polynomial and find its fourth-order difference. 2+3=5

2. (a) Write an algorithm for Newton-Raphson formula to find the root of algebraic equation. Further find the cube root of 10 using Newton-Raphson formula. 3+2=5

(b) If

$$y = \frac{1}{x(x+3)(x+6)}$$

then show that

$$\Delta^2 y = \frac{108}{x(x+3)(x+6)(x+9)(x+12)}$$

where Δ is the forward difference operator. 5

UNIT—II

3. (a) From the data given below, find the number of students whose weight is between 60 and 70 : 4

Weight	:	0–40	40–60	60–80	80–100	100–120
No. of Students	:	250	120	100	70	50

(b) Obtain Newton's divided difference interpolation formula for interpolation with non-equal intervals of the argument. 6

4\. (a) Use Lagrange's formula to find a polynomial of degree three which takes the values prescribed below and hence find the value of $f(x)$ when $x = 2$: 4+1=5

x	:	0	1	3	4
$f(x)$	:	−12	0	6	12

(b) The area A of a circle of diamater d is given for the following values :

d	:	80	85	90	95	100
A	:	5026	5674	6362	7088	7854

Find the approximate value for the area of a circle of diameter 97. 5

UNIT—III

5\. (a) Write an algorithm for Gauss elimination method. Further show the back substitution procedure in the Gaussian elimination method of solving a simultaneous linear equation. 2+3=5

(b) By Crout's method, solve the following system of simultaneous equations : 5

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

(Turn Over)

6. (a) What is diagonally dominant matrix? Make the following system of equations diagonally dominant and write the corresponding system of simultaneous equations : 2+3=5

$$3x+9y-2z=10$$
$$4x+2y+13z=19$$
$$4x-2y+z=3$$

(b) Solve the system of following linear equations by Gauss-Jordan method : 5

$$10x+y+z=12$$
$$2x+10y+z=13$$
$$x+y+5z=7$$

UNIT—IV

7. (a) Compute by Simpson's rule the value of the integral

$$I=\int_0^1 \frac{x^2}{1+x^2}\,dx$$

by dividing into four equal parts. 5

(b) Obtain the general quadrature formula for equidistant points to find the approximate integration of any function for which numerical values are known. 5

8. *(a)* Obtain the first and second derivatives of the function tabulated below, at the point $x = 0 \cdot 51$: 5

x :	0·4	0·5	0·6	0·7	0·8
y :	1·5836494	1·7974426	2·0442376	2·3275054	2·6510818

(b) Compute the value of the integral

$$I = \int_0^{\pi/3} \sqrt{\cos\theta}\, d\theta$$

by trapezoidal rule. 5

UNIT—V

9. *(a)* Solve

$$\frac{dy}{dx} = 1 - y, \; y(0) = 0$$

using Euler's method. Find y at $x = 0 \cdot 1$ and $x = 0 \cdot 2$. Compare the result with results of the exact solution. 3+2=5

(b) Using Taylor's method, find $y(0 \cdot 1)$ correct to 3-decimal places from

$$\frac{dy}{dx} + 2xy = 1, \; y_0 = 0$$

5

10. Using any Predictor-Corrector method, find $y(0 \cdot 4)$ for the differential equation

$$\frac{dy}{dx} = 1 + xy, \; y(0) = 2$$

10

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2015

(5th Semester)

MATHEMATICS

Paper : MATH–351

(Computer-oriented Numerical Analysis)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. By definition of backward difference operator, $\nabla f(x)$ equals to

(a) $f(x+h)-f(x)$ ☐

(b) $f(x+h)+f(x)$ ☐

(c) $f(x)-f(x+h)$ ☐

(d) $f(x)-f(x-h)$ ☐

2. If

$$f(x) = \frac{1}{(x+1)(x+2)(x+3)}$$

then the value of $\Delta^3 f(x)$ is

(a) $-60x^{(-6)}$ ☐

(b) $12x^{(-5)}$ ☐

(c) $360x^{(-7)}$ ☐

(d) $-12x^{(-5)}$ ☐

3. If $f(0) = 5$, $f(1) = 1$, $f(2) = 9$, $f(3) = 25$ and $f(4) = 55$, then the value of $f(5)$ is

(a) 105 ☐

(b) 115 ☐

(c) 125 ☐

(d) None of the above ☐

4. If $u_1 = 1$, $u_3 = 17$, $u_4 = 43$ and $u_5 = 89$, then the value of u_2 is

(a) 10 ☐

(b) 15 ☐

(c) 6 ☐

(d) 5 ☐

5. The method for obtaining the solution of the system of simultaneous equations by Gauss-Jordan elimination method depends on reducing the coefficient matrix to a/an

(a) lower triangular matrix ☐

(b) upper triangular matrix ☐

(c) diagonal matrix ☐

(d) diagonally dominant matrix ☐

6. The coefficient matrix obtained from the simultaneous equations

$$a_{11}x + a_{12}y + a_{13}z = d_1$$
$$b_{21}x + b_{22}y + b_{23}z = d_2$$
$$c_{31}x + c_{32}y + c_{33}z = d_3$$

will be a diagonally dominant matrix, if

(a) $|a_{11}| \geq |a_{12}| + |a_{13}|$, $|b_{21}| \geq |b_{22}| + |b_{23}|$, $|c_{31}| \geq |c_{32}| + |c_{33}|$ ☐

(b) $|a_{11}| \geq |a_{12}| + |a_{13}|$, $|b_{22}| \geq |b_{21}| + |b_{23}|$, $|c_{33}| \geq |c_{31}| + |c_{32}|$ ☐

(c) $|a_{11}| \leq |a_{12}| + |a_{13}|$, $|b_{22}| \leq |b_{21}| + |b_{23}|$, $|c_{33}| \leq |c_{31}| + |c_{32}|$ ☐

(d) $|a_{11}| + |a_{12}| + |a_{13}| \geq |d_1|$, $|b_{21}| + |b_{22}| + |b_{23}| \geq |d_2|$, $|c_{31}| + |c_{32}| + |c_{33}| \geq |d_3|$ ☐

7. In the general quadrature formula, Simpson's 1/3rd rule is obtained by putting

(a) $n = 1$ ☐

(b) $n = 4$ ☐

(c) $n = 2$ ☐

(d) $n = 2$ and 4 both ☐

8. When numerical integration is applied for the integration of a function of single variable, the method is called

(a) mechanical quadrature ☐

(b) general quadrature ☐

(c) Simpson's $\frac{1}{3}$rd rule ☐

(d) trapezoidal rule ☐

9. Taylor series method is a powerful single-step method if we are able to find easily the successive

(a) integration ☐

(b) derivatives ☐

(c) continuity ☐

(d) partial derivatives ☐

10. Which of the following statements is wrong?

(a) Modified Euler's method is Runge-Kutta method of second order. ☐

(b) Euler's method is the Rung-Kutta method of first order. ☐

(c) For solving ordinary differential equation numerically, the most reliable and most accurate method is Runge-Kutta method. ☐

(d) For solving ordinary differential equation numerically, Euler's method needs h to be very large to get a reasonable accuracy. ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. Obtain the relation $\nabla^n f(x+nh) = \Delta^n f(x)$, where Δ is the forward difference operator, and ∇ is the backward difference operator.

2. Show that the divided differences are independent of the order of arguments, i.e., $\delta(x_0, x_1) = \delta(x_1, x_0)$. Is it true for more than two arguments also?

3. Solve the given equations by Gauss elimination method :

$$x + y = 2;\ 2x + 3y = 5$$

4. Find the value of $\log 2^{1/3}$ from

$$\int_0^1 \frac{x^2}{1+x^3}\, dx$$

using Simpson's $\frac{1}{3}$rd rule with $h = 0 \cdot 25$.

5. Using Picard's method, solve the differential equation

$$\frac{dy}{dx} = 1 + xy \text{ with } y(0) = 2$$

and find $y(0 \cdot 2)$.

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